Online Appendices to "Inventory Commitment and Monetary Compensation under Competition"

Junfei Yu, Fuqiang Zhang, Renyu Zhang, Yugang Yu

Appendix A: Summary of Notations

Table 1 Summary of Notations

$R_i:$	Retailer i $(i=1,2)$
$p_i:$	Price of Retailer i
$q_i:$	Inventory stocking quantity of Retailer i
$\alpha_i:$	Market share/size of Retailer i
$\Pi_i:$	Expected profit of Retailer i
$D:$	Market aggregate demand
$s:$	Unit search cost
$c:$	Procurement cost
$v:$	Product valuation
$F(\cdot):$	Cumulative distribution function of demand; $\bar{F}(x):=1-F(x)$
$f(\cdot):$	Density function of demand distribution
$x:$	Customer location on the Hotelling line; $x\in\mathcal{M}$ and $\mathcal{M}=[0,1]$
$\mathbb{E}[\cdot]:$	Expectation operation
$x\wedge y:$	Minimum operation
$\theta_i:$	Customers' (rational) expectation of R_i's inventory availability probability

Appendix B: Deterministic Hotelling Model Benchmark

In this section, we introduce the classic Hotelling competition model with deterministic demand as the benchmark. The comparison between our base model and the deterministic benchmark could help us crystallize the impact of demand uncertainty and customers' availability concern.

We consider the same Hotelling line setup as the base model presented in Section 3.1 but with deterministic total market size. Specifically, we assume the aggregate market demand D is deterministic and known to everyone in the market. Without loss of generality, we normalize $D=\mu$. In the absence of demand uncertainty, the retailers will order exactly the amount of their respective market share, so every customer will be able to obtain her requested product. The two retailers R_1 and R_2 determine their respective prices p_1 and p_2 to maximize their own profits, whereas each retailer choose whether and where to visit. As in the base model, we focus on the equilibrium under competition. Let (p_d^*, q_d^*) be the equilibrium outcomes, where p_d^* is the equilibrium price and q_d^* is the equilibrium order quantity of each retailer. Similar to the base model in the main paper, it is straightforward to show that if s is small, R_1 and R_2 can serve the entire market, each covering 50% of the customers. If, otherwise, s is too large, there is essentially no competition between the two retailers and the market is not completely covered. Formally, we characterize the equilibrium prices (with competition) of the deterministic benchmark in the following lemma, which shows that the equilibrium price is increasing in s.

Lemma 3. *Assume that $D=\mu$ with certainty. If $s<\frac{2(v-c)}{3}$, $p_d^*=s+c$ and $q_d^*=\frac{\mu}{2}$. Each retailer covers 50% of the market.*

Proof of Lemma 3

In a deterministic Hotelling model, the retailers compete on market share by charging their prices. Demand is determined and public knowledge to all players in the market, so there is no issue of product availability. Without loss of generality, we shall use retailer R_1 as an example in the analysis.

When the search cost s is small, the two retailers cover the entire market. Given price p_1 and p_2, consumers located at $x \in [0,1]$ visit the retailer R_1 if $v - p_1 - sx \geq v - p_2 - s(1-x) \geq 0$. Thus, the retailer R_1 earns a market share $\frac{-p_1+p_2+s}{2s}$, and accordingly a profit $\pi_1(p_1) = (p_1 - c)\frac{-p_1+p_2+s}{2s}\mu$. Taking the first derivative of the profit function yields the retailer's best response function: $p_1^*(p_2) = \frac{p_2+s+c}{2}$. Since the two retailers are symmetric, retailer R_2 asks the same optimal price $p_2^*(p_1) = \frac{p_1+s+c}{2}$ to maximize his own profit. Note that the best response function, $p_i^*(p_{3-i})$, is increasing in price p_{3-i}, where $i \in \{1,2\}$, so there exists a unique equilibrium. In particular, the two retailers have the same optimal solutions in the equilibrium: $p_d^* = s + c$, $q_d^* = \frac{\mu}{2}$, and each covers half of the market. Finally, to guarantee $v - p_d^* - \frac{s}{2} > 0$, we obtain $s < \frac{2(v-c)}{3}$. $\square$

Appendix C: A Two-Stage Model with Customer Switching

In this section, we introduce a two-stage model with customer switching behavior upon stockout (i.e., without Assumption 1). There are two customer segments in the market: the switching customers (with proportion γ) and the non-switching customers (with proportion $1-\gamma$). In the first stage, both switching and non-switching customers visit their focal retailers and purchase the product if it is in stock. In the second stage, if the focal retailer is out of stock, the non-switching customers will directly leave the market, whereas the switching customers will switch to the other retailer for substitutes.

Customers' Problem. We first analyze the customers' problem in the second stage. Since the non-switching customers leave the market in the second stage, we only need to analyze the switching customers' decision problem. Consider a representative switching customer (at location x) who finds the product out of stock at her focal retailer, R_1 (resp. R_2). The customer would then switch to R_2 (resp. R_1) for a substitute in the second stage or leave the market. To avoid trivial analysis, we assume that the search cost, s, is small enough such that all switching customers will search for substitutes upon stockout (i.e., the switching customers earn a non-negative expected utility if switch to the competing retailers for substitutes). Therefore, the expected net surplus for the customer from switching to R_2 (resp. R_1) upon stockout at R_1 (resp. R_2) is: $U_{12}(x) = (v - p_2)\hat{\theta}_2(p_1,p_2) - s(1-x)$ (resp. $U_{21}(x) = (v-p_1)\hat{\theta}_1(p_1,p_2) - sx$), where $\hat{\theta}_i(p_1,p_2)$ is customers' belief on retailer R_i's inventory availability in the second stage.

We next examine the switching customers' choice of visiting the focal retailers by evaluating her expected utility in the first stage. For a switching customer at location x, her utility to visit R_1 (resp. R_2) with the product being available is $v - p_1 - sx$ (resp. $v - p_2 - s(1-x)$). Instead, if the product is out of stock, the customer switches to R_2 (resp. R_1) with an expected surplus $-sx + U_{12}(x)$ (resp. $-s(1-x) + U_{21}(x)$). Hence, the expected total utility of a switching customer located at x to visit R_1 (resp. R_2) in the first stage is $U_1(x) = (v-p_1)\theta_1(p_1,p_2) - sx + (1-\theta_1(p_1,p_2))U_{12}(x)$ (resp. $U_2(x) = (v-p_2)\theta_2(p_1,p_2) - s(1-x) + (1-\theta_2(p_1,p_2))U_{21}(x)$), where $\theta_i(p_1,p_2)$ is customers' belief on retailer R_i's inventory availability in the first stage. The customer chooses to first visit a focal retailer from which she can earn a higher total expected utility, i.e., $U_i(x) \geq U_{3-i}(x)$, and then switches to the competing retailer upon stockout. It is worth noting that the consumers' beliefs on retailer's inventory availability probabilities in two stages are different, $\theta_i(p_1,p_2) \neq \hat{\theta}_i(p_1,p_2)$, as the switching customers can update their beliefs upon stockout.

Finally, we examine the non-switching customer's choice of visiting the focal retailers. Similar to the base model, for a non-switching customer at location x, her expected utility to visit R_1 (resp. R_2) is $U_1(x) = (v-p_1)\theta_1(p_1,p_2) - sx$ (resp. $U_2(x) = (v-p_2)\theta_2(p_1,p_2) - s(1-x)$), where $\theta_i(p_1,p_2)$ is customers' belief on retailer R_i's inventory availability in the first stage. Note that the non-switching and switching customers' beliefs on retailer's inventory availability probability are the same in the first stage, as they arrive at the focal retailers at the same time.

To summarize, there exists a threshold for the switching customers:

$$x_s(p_1,p_2) = \frac{\theta_1(p_1,p_2)}{\theta_1(p_1,p_2)+\theta_2(p_1,p_2)}\Bigg(1 + \frac{(v-p_1)\left[\theta_1(p_1,p_2) - (1-\theta_2(p_1,p_2))\hat{\theta}_1(p_1,p_2)\right]}{s} - \frac{(v-p_2)\left[\theta_2(p_1,p_2) - (1-\theta_1(p_1,p_2))\hat{\theta}_2(p_1,p_2)\right]}{s}\Bigg),$$

such that a switching customer with location x will first patronize R_1 (resp. R_2) if $x \le x_s(p_1,p_2)$ (resp. $x > x_s(p_1,p_2)$) and then switch to the other retailer upon stockout. Moreover, there exists another threshold for the non-switching customers:

$$x(p_1,p_2) = \frac{1}{2} + \frac{(v-p_1)\theta_1(p_1,p_2) - (v-p_2)\theta_2(p_1,p_2)}{2s},$$

such that a non-switching customer with location x will patronize R_1 (resp. R_2) only if $x \le x(p_1,p_2)$ (resp. $x > x(p_1,p_2)$). The total market size for retailer R_1 (resp. R_2) is $\alpha_1(p_1,p_2) = \gamma(x_s(p_1,p_2) + (1-\theta_2(p_1,p_2))(1-x_s(p_1,p_2))) + (1-\gamma)x(p_1,p_2)$ (resp. $\alpha_2(p_1,p_2) = \gamma(1-x_s(p_1,p_2) + (1-\theta_1(p_1,p_2))x_s(p_1,p_2)) + (1-\gamma)(1-x(p_1,p_2)))$).

Retailer's Problem. We next analyze the retailer's pricing and inventory problem. The retailer satisfies demands from non-switching and switch customers in the first stage. In the second stage, the retailer satisfies the switching customers' demand via the remaining on-hand stock. Given market size $\alpha_i(p_1,p_2)$ defined above, retailer R_i's profit maximization problem is:

$$\max_{(p_i,q_i)} \{p_i \mathbb{E}(\alpha_i(p_1,p_2)D \wedge q_i) - cq_i\}.$$

Therefore, given a price p_i, the retailer R_i's optimal ordering strategy is the newsvendor solution: $q_i = \alpha_i(p_1,p_2)F^{-1}\left(\frac{p_i-c}{p_i}\right)$.

Consumers' Belief on Inventory Availability Probability. Next, we model the customers' beliefs on retailer's inventory availability probability, staring from the first stage. Similar to the analysis of the base model, conditioned on the existence of a customer, her belief about retailer R_i's demand y_i in the first stage is a random variable with probability density function $g_i(y_i|p_1,p_2) := \frac{y}{\tilde{\alpha}_i(p_1,p_2)\mu} f\left(\frac{y}{\tilde{\alpha}_i(p_1,p_2)}\right)$, where $\tilde{\alpha}_1(p_1,p_2) = \gamma x_s(p_1,p_2) + (1-\gamma)x(p_1,p_2)$ and $\tilde{\alpha}_2(p_1,p_2) = \gamma(1-x_s(p_1,p_2)) + (1-\gamma)(1-x(p_1,p_2))$. Note that $\tilde{\alpha}_i(p_1,p_2)$ represents retailer R_i's market size in the first stage (i.e., customers who choose retailer R_i as their focal retailer). Each customer holds an identical belief about the inventory availability for R_i so we have:

$$\theta_i(p_1,p_2) = \int_y \frac{\min\{q_i, \tilde{\alpha}_i y\}}{\tilde{\alpha}_i y} g_i(y|p_1,p_2)dy.$$

In the second stage, switching customers switch for substitutes if their focal retailers are out of stock. The competing retailers satisfy demand from the switching customers using the stock left from the first stage. For example, the demand for retailer R_i is $[\tilde{\alpha}_{3-i}(p_1,p_2)D - q_{3-i}]^+$ and his remaining stock is $[q_i - \tilde{\alpha}_i(p_1,p_2)D]^+$ in the second stage. Therefore, the customers' belief about retailer's inventory availability probability is

$$\hat{\theta}_i(p_1,p_2) = \int_y \frac{\min\{(\tilde{\alpha}_{3-i}(p_1,p_2)y - q_{3-i})^+, (q_i - \tilde{\alpha}_i(p_1,p_2)y)^+\}}{(\tilde{\alpha}_{3-i}(p_1,p_2)y - q_{3-i})^+} g(y|p_1,p_2)dy.$$

Now, we are ready to characterize the symmetric equilibrium price and inventory decisions of the retailers. The equilibrium price can be obtained through the following maximization problem:

$$\begin{aligned}
\max_{0\le p\le v} \quad & \Pi(p,p^*) = p\mathbb{E}\left[\alpha(p,p^*)D \wedge q(p,p^*)\right] - cq(p,p^*) \\
\text{s.t.} \quad & q(p,p^*) = \alpha(p,p^*)F^{-1}\left(\frac{p-c}{p}\right), \\
& \alpha(p,p^*) = \gamma\alpha_1(p,p^*) + (1-\gamma)\alpha_2(p,p^*), \\
& \alpha_1(p,p^*) = x_s(p,p^*) + (1-\theta(p,p^*))(1-x_s(p,p^*)), \\
& \alpha_2(p,p^*) = x(p,p^*),
\end{aligned}$$

where $\alpha_1(p,p^*)$ and $\alpha_2(p,p^*)$ represent market size from switching and non-switching customers, respectively.

Appendix D: Proof of Statements

Proof of Proposition 1

Given the equilibrium retailer decisions (p^*, q^*), a customer located at x has an expected payoff of $(v - p^*)\theta^*(p^*) - sx$, where $x \in [0,1]$. Note that, if the search cost s is small, the retailers compete on both price and inventory availability and the market $\mathcal{M}$ is fully covered under equilibrium. If the search cost s is large, $\mathcal{M}$ is not fully covered in equilibrium and, thus, the retailers do not directly compete with each other. In this case, the equilibrium outcome satisfies $(v-p^*)\theta^*(p^*) - s\alpha^* = 0$, where α^* is the equilibrium market share of a retailer. Hence, the expected payoff of the customers located at $x = \alpha^*$ and $x = 1-\alpha^*$ should be 0. Finally, when the search cost s is in a medium range, $\mathcal{M}$ is fully covered but the two retailers do not compete with each other. In this case, each retailer covers half of the market share under equilibrium. Thus, we have that $(v-p^*)\theta^*(p^*) - \frac{1}{2}s = 0$. For the rest of our proof, we use R_1 as the focal retailer and we shall focus on the first case where the two retailers compete with each other.

Let p be the price charged by retailer R_1 (the focal retailer), p' be the price charged by retailer R_2, α be the market share of R_1, and α' be the market share of R_2. Since the two retailers cover the entire market, a customer at the intersection of their respective market segments should be indifferent between visiting either retailer, i.e., $(v-p)\theta^*(p) - s\alpha = (v-p')\theta^*(p') - s(1-\alpha') \ge 0$. Recall that retailer R_2 charges price p', we next analyze R_1's best response function given price p', which will be denoted as $p^*(p')$. We write R_1's profit as $\Pi(p,p') := p\mathbb{E}(\alpha(p,p')D \wedge q^*(p,p')) - cq^*(p,p')$, where $q^*(p,p') = \alpha(p,p')F^{-1}(\frac{p-c}{p})$, and its market share $\alpha(p,p')$ satisfies the following equilibrium condition (the expected payoff to visit R_1 is the same as that to visit R_2): $(v-p)\theta^*(p) - s\alpha(p,p') = (v-p')\theta^*(p') - s(1-\alpha(p,p'))$. For simplicity, we rewrite the equilibrium condition as $U(p) - s\alpha(p,p') = U(p') - s(1-\alpha(p,p'))$, where $U(p) = (v-p)\theta^*(p)$. Therefore, for any given price p' from retailer R_2, the focal retailer's best price response satisfies $p^*(p')$, i.e.,

$$\begin{aligned}
p^*(p') :&= \arg\max_{0\le p\le v} \Pi(p,p') \\
&= \arg\max_{0\le p\le v} \left(\frac{1}{2} + \frac{(v-p)\theta^*(p) - (v-p')\theta^*(p')}{2s}\right)\left\{p\mathbb{E}\left[D \wedge F^{-1}\left(\frac{p-c}{p}\right)\right] - cF^{-1}\left(\frac{p-c}{p}\right)\right\}.
\end{aligned}$$

To find R_1's best response $p^*(p')$, we take derivative of the profit function $\Pi(p,p')$ with respect to price p, which yields

$$\frac{\partial \Pi(p,p')}{\partial p} = \frac{1}{2s}U'(p)\pi(p) + \left(\frac{1}{2} + \frac{U(p)-U(p')}{2s}\right)\pi'(p),$$

where $\pi(p) := p\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p-c}{p}\right)\right) - cF^{-1}\left(\frac{p-c}{p}\right)$.

According to Lemma 2, we know that $U(p)$ is decreasing and concave in p for $p \in [\hat{p}, v)$, where $\hat{p}$ maximizes $U(p)$. Moreover, we know that $U'(p) = 0$ at $p = \hat{p}$; $U'(p) < 0$ and $U(p) = 0$ at $p = v$. Since $\pi(p)$ is increasing in p, we have $\Pi'(p) > 0$ at $p = \hat{p}$ and $\Pi'(p) < 0$ at $p = v$. Hence, the first-order condition, $\Pi'(p) = 0$, results in a unique optimal price $p^*(p) \in [\hat{p}, v)$ when the search cost is sufficiently small. Furthermore, if the two retailers charge the same equilibrium price p^*, the equilibrium price satisfies the condition $U'(p^*)\pi(p^*) + s\pi'(p^*) = 0$.

Next, we prove the existence and uniqueness of the equilibrium. The implicit function theorem and the envelope theorem together yield $\frac{d^2p^*(p')}{d(p')^2} = \{-\frac{\partial}{\partial p}\frac{\partial^2\Pi(p,p')}{\partial p^2}\cdot\frac{\partial^2\Pi(p,p')}{\partial p\partial p'} + \frac{\partial^2\Pi(p,p')}{\partial p^2}\cdot\frac{\partial}{\partial p'}\frac{\partial^2\Pi(p,p')}{\partial p\partial p'}\}/(\frac{\partial^2\Pi(p,p')}{\partial p\partial p'})^2$. Thus, it can be easily verified that $\frac{\mathrm{d}p^*(p')}{\mathrm{d}p'} > 0$ and $\frac{\mathrm{d}^2p^*(p')}{\mathrm{d}(p')^2} < 0$, i.e., $p^*(p')$ is concavely increasing in p'. In addition, observe that $\lim_{p'\to v} p^*(p') < v$ and $\lim_{p'\to \hat{p}} p^*(p') \geq \hat{p}$. Thus, the function $p^*(p') - p'$ has a unique root on $[\hat{p}, v)$, which implies that the best-response function $p^*(p')$ has a unique fixed point on the interval $[\hat{p}, v)$. In other words, the equilibrium price p^* satisfies the equation $p^*(p^*) - p^* = 0$, which also implies that the equilibrium is symmetric. This proves the existence, uniqueness, and symmetry of the equilibrium. By the symmetry of the equilibrium outcome, we have $\alpha^* = \frac{1}{2}$ and $q^* = \frac{1}{2}F\left(\frac{p^*-c}{p^*}\right)$ under equilibrium. Finally, to complete the proof, we need to guarantee that the two retailers compete on market share under the equilibrium price p^*. That is, $U(p^*) \geq \frac{s}{2}$, where p^* is the equilibrium price characterized above. □

Proof of Proposition 2

For the Hotelling model with deterministic demand, the retailer's market share function is $\frac{-p+p_d^*+s}{2s}$ (see the proof of Lemma 3). Hence, the equilibrium price p_d^* can be obtained by the following first-order condition:

$$-\frac{1}{2s}(p-c)\mu + \frac{1}{2}\mu = 0.$$

Therefore, we obtain the equilibrium price $p_d^* = c + s$.

In our base model with demand uncertainty, we have $\theta(p) = \frac{1}{\mu}\int_0^\infty \left(y \wedge F^{-1}\left(\frac{p-c}{p}\right)\right) dF(y)$ and $\pi(p) = p\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p-c}{p}\right)\right) - cF^{-1}\left(\frac{p-c}{p}\right) = p\mu\theta(p) - cF^{-1}\left(\frac{p-c}{p}\right)$. Therefore, under equilibrium, the price satisfies the following first-order condition (also see the proof of Proposition 1):

$$\frac{-\theta(p) + (v-p)\theta'(p)}{2s}\pi(p) + \frac{1}{2}\mu\theta(p) = 0. \tag{5}$$

Clearly, we have $\theta(p) < 1$ and $\theta'(p) > 0$ for any $p \in [c, v)$. Moreover, we have $\pi(p) \leq (p-c)\mu$. The first-order condition (5) gives the equilibrium price p^*.

Next, we show $p^* > p_d^* = c + s$. Define

$$g(p) := \frac{-\theta(p) + (v-p)\theta'(p)}{2s}(p-c)\mu + \frac{1}{2}\mu\theta(p) \text{ and } g(\hat{p}^*) = 0.$$

Hence, $\hat{p}^* = c + s\frac{\theta(\hat{p}^*)}{\theta(\hat{p}^*)-(v-\hat{p}^*)\theta'(\hat{p}^*)} > c + s = p_d^*$. Recall that $\pi(p) \leq (p-c)\mu$, so (5) implies that $p^* > \hat{p}^* > p_d^*$, which concludes the proof. □

Proof of Proposition 3

We first examine the case when $\gamma = 1$ (i.e., all customers are switching customers). Assuming that the competing retailer charges the equilibrium price p_s^*, we know the focal retailer's (i.e., retailer R_1) market size is:

$$\alpha(p_1, p_s^*) = \frac{\theta^*(p_1)\theta^*(p_s^*)}{\theta^*(p_1)+\theta^*(p_s^*)}\left(1+\frac{(p_s^*-p_1)\theta^*(p_s^*)}{s}\right)+\min\left(\frac{(v-p_1)\theta^*(p_1)}{s},1\right)(1-\theta^*(p_s^*)).$$

Let $\phi_1(p_1,p_s^*) := \frac{\theta^*(p_1)\theta^*(p_s^*)}{\theta^*(p_1)+\theta^*(p_s^*)}$ and $\phi_2(p_1,p_s^*) := \left(1+\frac{(p_s^*-p_1)\theta^*(p_s^*)}{s}\right)$, we have

$$\frac{d\phi_1(p_1,p_s^*)}{dp_1}\frac{(\theta^*(p_s^*))^2}{(\theta^*(p_1)+\theta^*(p_s^*))^2}\frac{d\theta^*(p_1)}{dp_1} > 0$$

$$\frac{d^2\phi_1(p_1,p_s^*)}{dp_1^2} = \frac{(\theta^*(p_s^*))^2}{(\theta^*(p_1)+\theta^*(p_s^*))^3}\left\{\frac{d^2\theta^*(p_1)}{dp_1^2}[\theta^*(p_s^*)+\theta^*(p_1)]-2\left(\frac{d\theta^*(p_1)}{dp_1}\right)^2\right\} < 0,$$

as $\frac{d^2\theta^*(p_1)}{dp_1^2} < 0$. Similarly, we can also obtain $\frac{d\phi_2(p_1,p_s^*)}{dp_1} < 0$ and $\frac{d^2\phi_2(p_1,p_s^*)}{dp_1^2} = 0$. Therefore, the term $\phi_1(p_1,p_s^*)\phi_2(p_1,p_s^*)$ is concave in p_1.

Now, we study the second term of the market size function. Let $\phi_3(p_1) = \frac{(v-p_1)\theta^*(p_1)}{s}$. As shown in Proposition 1, $\phi_3(p_1)$ is concave in p_1. Hence, the retailer's market size:

$$\alpha(p_1,p_s^*) = \phi_1(p_1,p_s^*)\phi_2(p_1,p_s^*)+\min(1,\phi_3(p_1))(1-\theta^*(p_s^*)),$$

is concave in price p_1.

Next, we examine a more general case when a fraction γ of the customers are switching customers and the rest, $1-\gamma$, are no-switching customers. Note that R_1's market size from the non-switch customers

$$\alpha(p_1,p_s^*) = (1-\gamma)\left\{\frac{1}{2}+\frac{(v-p)\theta(p)-(v-p_s^*)\theta^*(p_s^*)}{2s}\right\}$$

is convex in price p_1. Thus, the retailer's total market size from the switching customers and non-switching customers is convex in price p_1. Similar to the proof of Proposition 1, the retailer's profit function can be written as

$$\Pi(p_1,p_s^*) = \alpha(p_1,p_s^*)\left\{p\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p-c}{p}\right)\right]-cF^{-1}\left(\frac{p-c}{p}\right)\right\},$$

where $\alpha_1(p_1,p_s^*)$ represents the total market size in the presence of both switching customers and non-switching customers.

Since s is sufficiently small so that all switching customers will switch upon stockout, we have $(v-p_s^*)\theta^*(p_s^*) > s$. In this case, the equilibrium price satisfies the first-order condition as follows:

$$\left\{\gamma\left(\frac{1}{4}\frac{d\theta^*(p_s^*)}{dp_s^*}-\frac{(\theta^*(p_s^*))^2}{2s}\right)+\frac{(1-\gamma)U'(p_s^*)}{2s}\right\}\pi(p_s^*)+\left\{\gamma\left(1-\frac{\theta^*(p_s^*)}{2}\right)+\frac{1-\gamma}{2}\right\}\pi'(p_s^*) = 0.$$

Similar to the proof of Proposition 1, when the search cost s is sufficiently small, the symmetric equilibrium price p_s^* will the unique root of the above first-order conditions. This concludes the proof. $\square$

Proof of Proposition 4

Similar to the other proofs, we set R_1 as the focal retailer and focus on the case where s is sufficiently small so that all switching customers will switch to the other retailer for substitutes upon stockout.

Assume that retailer R_2 charges the equilibrium price p_v^* and stocks the equilibrium inventory quantity q_v^*. The focal retailer R_1 maximizes his profit $\Pi(p,q) := p\mathbb{E}(\alpha(p,q)D \wedge q) - cq$. First of all, we drive the equilibrium condition of the market size given that all switching customers will switch upon stocks out. Recall that R_1's market size is

$$\alpha(p,q) = \gamma\left\{x_s(p,q) + [1 - x_s(p,q)]\,(1-\theta_v)\right\} + (1-\gamma)x(p,q),$$

where $x_s(p,q) = \frac{\theta}{\theta+\theta_v}\left\{\frac{(p_v^*-p)\theta_v}{s}+1\right\}$, $x(p,q) = \frac{1}{2} + \frac{(v-p)\theta-(v-p_v^*)\theta_v}{2s}$, $\theta = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \frac{q}{\alpha(p,q)}\right)\mathrm{d}F(y)$, $\theta_v(p,q) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \frac{q^*}{\alpha_v}\right)f(y)\mathrm{d}y$, and $\alpha(p,q)+\alpha_v(p,q) = 1+\gamma[1-\theta_v+x_s(p,q)(\theta_v-\theta)]$. Clearly, the market size function is decreasing in price p, i.e., $\frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}p} < 0$, and increasing in quantity q, i.e., $\frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}q} > 0$.

Then, we take the derivative of the profit function, $\Pi_v(p,q)$, with respect to q. The first-order condition implies that

$$q_v^*(p) = \alpha(p, q_v^*(p))F^{-1}\left(\frac{p-c}{p} + \frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}q}\bigg|_{q=q_v^*(p)}\int_0^{\frac{q_v^*(p)}{\alpha(p,q_v^*(p))}} x dF(x)\right).$$

Under equilibrium, we have $p = p_v^*,\ q = q_v^*(p^*) = \alpha(p_v^*, q_v^*)F^{-1}\left(\frac{p_v^*-c}{p_v^*} + \frac{\mathrm{d}\alpha(p_v^*,q_v^*)}{\mathrm{d}q_v^*}\int_0^{\frac{q_v^*}{\alpha(p_v^*,q_v^*)}} x\mathrm{d}F(x)\right)$, where $\alpha(p_v^*, q_v^*) = \gamma\left\{1 - \frac{1}{2}\theta^*\left(\frac{q_v^*}{\alpha(p_v^*,q_v^*)}\right)\right\} + \frac{1-\gamma}{2}$.

Comparing the equilibrium order quantity in the focal model, $q^*(p)/\alpha(p) = F^{-1}(\frac{p-c}{p})$, and the equilibrium order quantity in the model with inventory commitment, $q^*(p)/\alpha(p,q^*(p)) = F^{-1}\left(\frac{p-c}{p} + \frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}q}\big|_{q=q^*(p)}\int_0^{\frac{q^*(p)}{\alpha(p,q^*(p))}} x\mathrm{d}F(x)\right) := g_v(p)$, we find that $g_v(p)$ shares the same functional properties as $F^{-1}(\frac{p-c}{p})$, which is concavely increasing in p. Moreover, given the same price as in the focal model, the retailer in the inventory commitment model has a tendency to increase inventory stock.

Next, we examine the equilibrium price given the optimal quantity decision $q^*(p)$ following the path of symmetric equilibrium. Given R_2's decision, $(p, q^*(p))$, R_1 maximizes his expected profit:

$$\Pi(p) = p\mathbb{E}[(\alpha(p)D) \wedge q^*(p)] - cq^*(p),$$

subject to:

$$\alpha(p) = \gamma\left\{x_s(p) + (1-x_s(p))\left(1 - \theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)\right)\right\} + (1-\gamma)x(p),$$

$$x_s(p) = \frac{\theta\left(\frac{q^*(p)}{\alpha(p)}\right)}{\theta\left(\frac{q^*(p)}{\alpha(p)}\right) + \theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)}\left(\frac{p^*-p}{s}\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right) + 1\right) + \left(1 - \theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)\right),$$

$$x(p) = \frac{1}{2} + \frac{1}{2s}\left((v-p)\theta\left(\frac{q^*(p)}{\alpha(p)}\right) - (v-p^*)\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)\right),$$

$$\alpha_v(p) = 1 - \alpha(p) + \gamma\left\{1 - \theta\left(\frac{q^*(p)}{\alpha_v(p)}\right) + x_s(p)\left(\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right) - \theta\left(\frac{q^*(p)}{\alpha(p)}\right)\right)\right\}.$$

To conclude this proof, we need to argue the uniqueness of the symmetric equilibrium price p^* when the search cost s is sufficiently small. Due to the complexity of the problem, we examine the first-order condition that determines the equilibrium price as follows:

$$\Pi'(p) = \alpha'(p)\pi(p) + \alpha(p)\pi'(p) = 0,$$

where $\pi(p) = p\mathbb{E}\left(D \wedge \frac{q^*(p)}{\alpha(p)}\right) - c\frac{q^*(p)}{\alpha(p)}$. Following the same argument as in the proof of Proposition 1, we have $\pi'(p) > 0$, $\alpha'(p) \propto \frac{1}{s}$, and $\alpha'' \propto \frac{1}{s}$. As a result, when the search cost s is sufficiently small, there is a unique solution of the first-order condition and we denote the equilibrium price as p_v^*. Putting everything together, we know there is a unique symmetric equilibrium $(p_v^*, q_v^*, \alpha_v^*)$. □

Proof of Lemma 1

To compare the profit of R_1 under different strategies, we first calculate his profit in different circumstances. We first examine the case where R_2 does not reveal his inventory information. In this case, a customer at the purchasing threshold forms a belief $\theta_2 = \theta^*(p_2) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}(\frac{p_2-c}{p_2})\right) f(y)\mathrm{d}y$ of the inventory availability probability at R_2. A customer will visit R_1 first if and only if her expected utility of visiting R_1 dominates that of visiting R_2. Therefore, if R_1 also does not reveal its inventory information to the market, his market size α_1 is given by

$$\begin{aligned}\alpha_1(p_1) =& \gamma\left\{\frac{\theta_1^*(p_1)\theta_2^*(p_2)}{\theta_1^*(p_1)+\theta_2^*(p_2)}\left(1+\frac{\theta_2^*(p_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2))\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta^*(p_1)-(v-p_2)\theta^*(p_2)}{2s}\right\}.\end{aligned} \tag{6}$$

Thus, the maximum profit of R_1 if he does not adopt the inventory commitment strategy is

$$\Pi_{d,d} := \max_{0\le p_1\le v}\left\{\alpha_1(p_1)\cdot\left\{p_1\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p_1-c}{p_1}\right)\right]-cF^{-1}\left(\frac{p_1-c}{p_1}\right)\right]\right\}.$$

Similarly, if R_1 adopts the inventory commitment strategy, his market share α_1 satisfies the following equation:

$$\begin{aligned}\alpha_1(p_1,q_1) =& \gamma\left\{\frac{\theta_1^*(p_1,q_1)\theta_2^*(p_2)}{\theta_1^*(p_1,q_1)+\theta_2^*(p_2)}\left(1+\frac{\theta_2^*(p_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2))\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta_1^*(p_1,q_1)-(v-p_2)\theta_2^*(p_2)}{2s}\right\},\end{aligned} \tag{7}$$

where $\theta_1^*(p_1,q_1) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_1}{\alpha_1}\right)f(y)\mathrm{d}y$ and $\theta_2^*(p_2) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}(\frac{p_2-c}{p_2})\right)f(y)\mathrm{d}y$. Therefore, the maximum profit of R_1 if he adopts the inventory commitment strategy is

$$\Pi_{v,d} := \max_{0\le p_1\le v, q_1\ge 0}\left\{p_1\mathbb{E}[\alpha_1(p_1,q_1)D\wedge q_1]-cq_1\right\},$$

where α_1 satisfies equation (7).

We now turn our attention to the case where R_2 adopts the inventory commitment strategy. If R_1 does not reveal his inventory information to the market, customers form a belief $\theta_1 = \theta^*(p_1) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}(\frac{p_1-c}{p_1})\right)f(y)\mathrm{d}y$ about the inventory availability probability of the retailer. Therefore, the market size α_1 is

$$\begin{aligned}\alpha_1(p_1) =& \gamma\left\{\frac{\theta_1^*(p_1)\theta_2^*(p_2,q_2)}{\theta_1^*(p_1)+\theta_2^*(p_2,q_2)}\left(1+\frac{\theta_2^*(p_2,q_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2,q_2))\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta_1^*(p_1)-(v-p_2)\theta_2^*(p_2,q_2)}{2s}\right\},\end{aligned} \tag{8}$$

where $\theta_2^*(p_2,q_2)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_2}{\alpha_2}\right)f(y)\mathrm{d}y$. The maximum profit of R_1 is

$$\Pi_{d,v}:=\max_{0\le p_1\le v}\left\{\alpha_1(p_1)\cdot\left\{p_1\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p_1-c}{p_1}\right)\right]-c\bar{F}^{-1}\left(\frac{c}{p_1}\right)\right]\right\}.$$

If R_1 adopts the inventory commitment strategy, his market share α_1 satisfies the following equation:

$$\begin{aligned}\alpha_1(p_1,q_1)=&\gamma\left\{\frac{\theta_1^*(p_1,q_1)\theta_2^*(p_2,q_2)}{\theta_1^*(p_1,q_1)+\theta_2^*(p_2,q_2)}\left(1+\frac{\theta_2^*(p_2,q_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2,q_2))\right\}\\&+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta_1^*(p_1,q_1)-(v-p_2)\theta_2^*(p_2,q_2)}{2s}\right\},\end{aligned}\tag{9}$$

where $\theta_1^*(p_1,q_1)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_1}{\alpha_1}\right)f(y)\mathrm{d}y$ and $\theta_2^*(p_2,q_2)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_2}{\alpha_2}\right)f(y)\mathrm{d}y$. The maximum profit of R_1 if he adopts the inventory commitment strategy is

$$\Pi_{v,v}:=\max_{0\le p_1\le v,q_1\ge 0}\left\{p_1\mathbb{E}[\alpha_1(p_1,q_1)D\wedge q_1]-cq_1\right\}.$$

By comparing the profit functions of R_1 under different strategy profiles, it is straightforward to observe that the equilibrium market share of R_1 is larger if he commits to an inventory order quantity, regardless of whether R_2 reveals his inventory information. Hence, the profit of R_1 will be higher under the inventory commitment strategy if the retailer commits to an inventory level that leads to the same in-stock probability. Therefore, regardless of the price and order quantity decisions for R_2 and regardless of whether R_2 adopts the inventory commitment strategy, the profit of R_1 is higher if he adopts the inventory commitment strategy, i.e., $\Pi_{v,d}>\Pi_{d,d}$ and $\Pi_{v,v}>\Pi_{d,v}$. □

Proof of Proposition 5

First, we prove $\Pi_v^*\ge\Pi^*$ when there is no competition (i.e., s is sufficiently large). In the monopoly model (without commitment), a retailer's profit function is

$$\Pi(p)=p\mathbb{E}(\alpha(p)D\wedge q^*(p))-cq^*(p),$$

where $q^*(p)=\alpha(p)F^{-1}(\frac{p-c}{p})$, $\alpha(p)=\frac{v-p}{s}\theta^*(p)$ and $\theta^*(p)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}\left(\frac{p-c}{p}\right)\right)f(y)\mathrm{d}y$. In the model with inventory commitment, a retailer's profit function is

$$\Pi_v(p,q)=p\mathbb{E}(\alpha(p,q)D\wedge q)-cq,$$

where $\alpha(p,q)=\frac{v-p}{s}\theta(p,q)$ and $\theta(p,q)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q}{\alpha(p,q)}\right)f(y)\mathrm{d}y$. It is clear from the profit functions that, the two models have the same profit functions but the model without inventory commitment has an additional constraint $q=\alpha(p)F^{-1}(\frac{p-c}{p})$. Hence, $\Pi_v^*=\max_{(p,q)}\Pi_v(p,q)\ge\max_v\Pi(p,q(p))=\max\Pi(p)=\Pi^*$. Therefore, if the search cost s is large such that the market is partially covered by the two retailers, we have $\Pi_v^*\ge\Pi^*$.

Now we turn to the case of full market coverage with customer switching. To begin with, we analyze the equilibrium pricing policies of both models when $s=0$, starting with the focal model. First, we examine the purchase decision of non-switching customers. When $s=0$, R_1 attracts demand from all non-switching customers if

$$(v-p_1)\theta^*(p_1)\ge(v-p_2)\theta^*(p_2).$$

Since the expected utility function, $(v-p)\theta^*(p)$, is concave in price p, to attract demand from all non-switching customers, the two retailers compete on offering lower prices when $p > \hat{p}$ and on offering higher prices when $p \leq \hat{p}$, where $\hat{p} = \max_p (v-p)\theta^*(p)$.

Next, we examine the purchase decision of switching customers when $s=0$. Consider a switching customer located at x, he will visit R_1 first if

$$(v-p_1)\theta^*(p_1) + (v-p_2)(1-\theta^*(p_1))\theta^*(p_2) \geq (v-p_2)\theta^*(p_2) + (v-p_1)(1-\theta^*(p_2))\theta^*(p_1).$$

Simplifying the above condition, we obtain $p_1 \leq p_2$. That is, if $p_1 < p_2$, R_1's market size from the switching customers is 1; otherwise, if $p_1 > p_2$, R_1's market size from the switching customers is $1-\theta(p_2)$. Hence, to compete for the switching customers, the retailers will compete on offering lower prices until $p = c$.

Now, we analyze the equilibrium price, p^*, for the general case where the market consists of γ portion of switching customers and $1-\gamma$ non-switching customers. First, the equilibrium price should be no greater than $\hat{p}$ ($p^* \leq \hat{p}$), as a lower price signals a higher market size thus a higher profit (for example, the retailers will compete on offering lower prices). Second, given that R_2 charges the price $\hat{p}$, R_1's market size is $\alpha(\hat{p},\hat{p}) = \gamma\left(\frac{1}{2} + \frac{1}{2}(1-\theta(\hat{p}))\right) + (1-\gamma)\frac{1}{2}$ if he charges the price $p = \hat{p}$. However, if R_1 decreases price to $p = \hat{p} - \epsilon$, his market size will be $\alpha(\hat{p}-\epsilon,\hat{p}) = \gamma$ (all the switching customers will visit R_1, while all the non-switching customers will vist R_2.). Therefore, the equilibrium price will be $\tilde{p}^* = \hat{p}$ when $\alpha(\hat{p},\hat{p}) \geq \alpha(\hat{p}-\epsilon,\hat{p})$, which gives the condition $\hat{p} \leq \theta^{-1}\left(\frac{1-\gamma}{\gamma}\right)$. Finally, we examine the region where R_2 charges a price $p_2 < \hat{p}$. Similarly, R_1's market size is $\alpha(p_2,p_2) = \gamma\left(\frac{1}{2} + \frac{1}{2}(1-\theta(p_2))\right) + (1-\gamma)\frac{1}{2}$ if he charges the same price $p = p_2$. However, if R_1 decreases price to $p_2 - \epsilon$, R_1's the market size is $\alpha(p_2-\epsilon,p_2) = \gamma$. Clearly, the equilibrium price is the one that gives a zero marginal increase in market size. Hence, we have $p^* = \theta^{-1}\left(\frac{1-\gamma}{\gamma}\right)$. Combining all the cases above, we have

$$\tilde{p}^* = \min\left\{\hat{p}, (\theta^*)^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right\},$$

where $\hat{p} = \max_p (v-p)\theta^*(p)$. Since $\tilde{p}^* > c$, we have $\Pi^* > 0$ in the focal model when $s=0$.

We now consider the model with inventory commitment given $s=0$. According to the first-order condition with respect to q, the retailer's optimal order quantity $q_v^*(p)$ satisfies the equation $q_v^*(p) = \alpha_v^* F^{-1}\left(\frac{p-c}{p} + \frac{d\alpha(p,q)}{dq}|_{q=q_v^*(p)} \int_0^{\frac{q_v^*(p)}{\alpha_v^*}} xf(x)dx\right)$. Since we have $\frac{d\alpha(p,q)}{dq}|_{q=q_v^*(p)} > 0$, for any given price p, the retailer tends to stock more under inventory commitment, i.e., $q_v^*(p) \geq q^*(p)$.

Similar to the analysis of the equilibrium price in the focal model when $s=0$, we have:

$$p_v^* = \min\left\{\hat{p}_v, (\theta_v^*)^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right\},$$

where $\hat{p}_v = \max_p (v-p)\theta_v^*(p, q^*(p))$. Clearly, we have $p_v^* \leq p^*$ since the same price in the model with inventory commitment signals a higher quantity and thus a higher inventory availability in the equilibrium. Further, note that if $c \to 0$, the optimal inventory availability in the model of inventory commitment is no less than that in the base model, $\theta_v^* \geq \theta^*$. Therefore, we have

$$\begin{aligned}
\Pi^* &= p^*\mathbb{E}\left(\alpha^* D \wedge q^*\right) - cq^* \\
&= \alpha^* \left\{ p^*\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p^*-c}{p^*}\right)\right) - cF^{-1}\left(\frac{p^*-c}{p^*}\right)\right\} \\
&\geq \alpha_v^* \left\{ p^*\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p^*-c}{p^*}\right)\right) - cF^{-1}\left(\frac{p^*-c}{p^*}\right)\right\} \\
&\geq \alpha_v^* \left\{ p_v^*\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)\right) - cF^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)\right\} \\
&\geq p_v^*\mathbb{E}\left(\alpha_v^* D \wedge q_v^*\right) - cq_v^* = \Pi_v^*.
\end{aligned}$$

The first inequality follows that $\alpha^* = \gamma\left(\frac{1}{2} + \frac{1}{2}(1-\theta^*)\right) + (1-\gamma)\frac{1}{2} \geq \gamma\left(\frac{1}{2} + \frac{1}{2}(1-\theta_v^*)\right) + (1-\gamma)\frac{1}{2} = \alpha_v^*$ as $\theta^* \leq \theta_v^*$. The second inequality follows from $p^* \geq p_v^*$. The third inequality follows from $q_v^* \neq \alpha_v^* F^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)$ (where the quantity $q_v^* = \alpha_v^* F^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)$ is the optimal quantity that maximizes the profit function given the price p_v^*). Therefore, the inventory commitment strategy results in a lower profit when $s = 0$ and $c \to 0$.

Finally, the two equilibrium profits, Π^* and Π_v^*, are both continuous in s and c. Moreover, we have just shown that $\lim_{s,c\to 0} \Pi^* > \lim_{s,c\to 0} \Pi_v^*$. Thus, there exist two thresholds $\bar{s}_v$ (for s) and $\bar{c}_v$ (for c), such that $\Pi_v^* < \Pi^*$ if $s < \bar{s}_v$ and $c < \bar{c}_v$. □

Proof of Proposition 6

We continue to use retailer R_1 as the focal retailer. Analogous to the proof of Proposition 1, we will focus on the case when the unit travel cost s is small such that the retailers compete with each other with full customer switching in equilibrium.

Given the equilibrium price p^* and equilibrium compensation m^*, the expected profit of retailer R_1 is

$$\Pi(p_1, m_1) = (p_1 + m_1)\mathbb{E}(\alpha_1 D \wedge q^*(p_1 + m_1)) - cq^*(p + m_1) - \alpha_1 m_1 \mu,$$

where $q^*(p_1 + m_1) = \alpha_1 F^{-1}(\frac{p_1+m_1-c}{p_1+m_1})$ and $\mu = \mathbb{E}(D)$. The market size is

$$\begin{aligned}
\alpha_1 =& \gamma\left\{\frac{\theta_1\theta^*}{\theta_1+\theta^*}\left(\frac{(p^*+m^*)-(p_1+m_1)}{s}\theta^* + 1\right) + \frac{\theta^*(m_1\theta^* - m^*\theta_1)}{s(\theta_1+\theta^*)} + (1-\theta^*)\right\} \\
&+ (1-\gamma)\left\{\frac{1}{2} + \frac{[v-(p_1+m_1)]\theta_1 - [v-(p^*+m^*)]\theta^* + (m_1-m^*)}{2s}\right\},
\end{aligned}$$

where $\theta = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p+m-c}{p+m}\right)\right)\mathrm{d}F(y)$. For ease of exposition, we define $t_1 = p_1 + m_1$, which refers to the effective marginal revenue of the product. Hence, the problem can be rewritten as

$$\begin{aligned}
&\Pi(t_1, m_1) = \alpha_1\left\{t_1\mathbb{E}\left(D \wedge F^{-1}\left(\frac{t_1-c}{t_1}\right)\right) - cF^{-1}\left(\frac{t_1-c}{t_1}\right) - m_1\mu\right\} \\
s.t.\quad &\alpha_1(t_1, m_1) = \gamma\left\{\frac{\theta_1\theta^*}{\theta_1+\theta^*}\left(\frac{t^*-t_1}{s}\theta^* + 1\right) + \frac{\theta^*(m_1\theta^* - m^*\theta_1)}{s(\theta_1+\theta^*)} + (1-\theta^*)\right\} \\
&\qquad + (1-\gamma)\left\{\frac{1}{2} + \frac{(v-t_1)\theta_1 - (v-t^*)\theta^* + (m_1-m^*)}{2s}\right\},
\end{aligned}$$

where $\theta = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{t-c}{t}\right)\right)\mathrm{d}F(y)$. It can be shown that the market size function is concave in t_1. Let $\pi(t_1, m_1) = t_1\mathbb{E}\left(D \wedge F^{-1}\left(\frac{t_1-c}{t_1}\right)\right) - cF^{-1}\left(\frac{t_1-c}{t_1}\right) - m_1\mu$. Following the path of symmetric equilibrium and the same argument in the proof of Proposition 1, when the search cost is sufficiently small, we can show that the expected profit of R_1 is concave in t_1, which further implies a unique equilibrium t^*. Given t^*, we next examine the retailer's best compensation response. Since $\pi(t^*, m_1)$ is linearly decreasing in m_1 and the market size is linearly increasing in m_1, the expected profit of R_1 is concave in m_1. Hence, we have a unique best compensation response $m^*(t^*)$. Therefore, we obtain a unique symmetric equilibrium (p_c^*, m_c^*). □

Proof of Proposition 7

We start the proof by verifying two extreme cases. We first consider the case when the search cost is zero (i.e., $s=0$). In this case, the two retailers compete on offering higher consumer expected payoff, because $\alpha'(p,m) \to -\infty$. For example, the expected payoff is $U(p_1+m_1)+m_1-sx$ for a non-switching customer located at $x \in [0,1]$ who chooses to visit R_1. The first term, $U(p_1+m_1)$, is concave with its maximum value, $U(\hat{p})$, at $p_1+m_1=\hat{p}$. The second term is linearly increasing in m_1. Similarly, the expected payoff is $U(p_1+m_1)+m_1-sx+(U(p_2+m_2)+m_2-s(1-x))(1-\theta_1)$ for a switching customer located at $x \in [0,1]$ who visits R_1 first. According to the expected utility of customers (switching customers and non-switching customers), retailers can always capture the entire market by continuously increasing compensation m. However, each retailer's profit function is strictly decreasing in compensation m, so the retailers have to stop raising compensation at zero profit. Therefore, each retailer obtains zero profit under equilibrium when $s=0$. In contrast, each retailer charges price $p=p^*$ in the base model, as shown in the proof of Proposition 5. Since we always have $p^*>c$, each retailer must have a positive profit in the base model. Therefore, we have $\Pi^* > \Pi_c^*$ when $s=0$.

We next examine the case when the search cost is large. In this case, the two retailers have no direct competition (i.e., partial market coverage). In the model with monetary compensation, each retailer maximizes his profit $\alpha(p+m)\left\{(p+m)\mathbb{E}(D \wedge F^{-1}(\frac{p+m-c}{p+m}))) - cF^{-1}(\frac{p+m-c}{p+m}) - m\mathbb{E}(D)\right\}$, where $\alpha(p+m)=\frac{U(p+m)}{s}$. In the base model, each retailer maximizes its profit $\alpha(p)\left\{(p)\mathbb{E}(D \wedge F^{-1}(\frac{p-c}{p})) - cF^{-1}(\frac{p-c}{p})\right\}$, where $\alpha(p)=\frac{U(p)}{s}$. The profit function in the model of monetary compensation restores to the profit function in the base model when $m=0$. Since m is a free variable, the base model is a special case of the monetary compensation model when $s=0$. In other words,

$$\Pi_c^* = \max_{(p,m)} \Pi_c(p,m) \geq \max_p \Pi_c(p,0) = \max_p \Pi(p) = \Pi^*.$$

Therefore, we have $\Pi_c^* \geq \Pi^*$ when $s \to \infty$.

Finally, recall that Π^* and Π_c^* are continuous in s. We have already obtained that $\Pi^* > \Pi_c^*$ when $s=0$; and that $\Pi_c^* \geq \Pi^*$ when $s \to \infty$. Therefore, there exists a threshold $\bar{s}_c$ such that $\Pi_c^* < \Pi^*$ if $s < \bar{s}_c$. □

Proof of Lemma 2

We first show that the customer's expected payoff function is concave in price p. We start by examining the average surplus for non-switching customers, $CS_1(p)=(v-p)\theta^*(p)-\frac{s}{4}$, where $\theta^*(p)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p-c}{p}\right)\right)f(y)\mathrm{d}y$. We have $\frac{d^2 CS_1(p)}{dp^2} = -2\frac{d\theta^*(p)}{dp} + (v-p)\frac{d^2\theta^*(p)}{dp^2}$. Clearly, if $\frac{d^2\theta^*(p)}{dp^2}<0$, then $CS_1(p)$ is concave in price p. Note that

$$\mu\frac{d\theta^*(p)}{dp} = \frac{\frac{c}{p}}{f\left(F^{-1}\left(\frac{p_i-c}{p_i}\right)\right)}\frac{c}{p^2} = \frac{1-F\left(F^{-1}\left(\frac{p_i-c}{p_i}\right)\right)}{f\left(F^{-1}\left(\frac{p_i-c}{p_i}\right)\right)}\frac{c}{p^2},$$

which is strictly decreasing in p given that D follows a distribution with an increasing failure rate. Hence, $\theta^*(p)$ is concave in p and thus, the average surplus for non-switching customers is also concave in p.

Next, we examine the average surplus for switching customers, $CS_2(p) = (v-p)(2-\theta^*(p))\theta^*(p) - s(1-\frac{3}{4}\theta^*(p))$. Clearly, the second term, $-s[1-(1-x)\theta^*(p)]$, is concave in p. Hence, the concavity of surplus function $CS_2(p)$ boils down to the concavity of term $(v-p)(2-\theta^*(p))\theta^*(p)$. Taking the second derivative of the term yields:

$$\frac{d^2}{dp^2}[(v-p)(2-\theta^*(p))\theta^*(p)] = -2p\frac{dg(\theta^*)}{d\theta^*}\frac{d\theta^*(p)}{dp} + (v-p)\left(\frac{d^2g(\theta^*)}{d(\theta^*)^2}\frac{d\theta^*(p)}{dp} + \frac{d\theta^*(p)}{dp}\frac{d^2\theta^*(p)}{dp^2}\right),$$

where $g(\theta^*) = (2-\theta^*(p))\theta^*(p)$ and $\theta^*(p) \in [0,1]$. Since $\frac{dg(\theta^*)}{d\theta^*} > 0$ and $\frac{d^2g(\theta^*)}{d(\theta^*)^2} < 0$, the first term is concave in price p. Thus, the average surplus for switching customers is also concave in price p. Finally, recall that the total average customers' surplus is a weighted summation of the average surplus functions of the two customer segments, therefore, the total average customers' surplus, $CS(p)$, is concave in price p. From $CS'(p) = 0$, the expected payoff function $CS(p)$ is maximized at $p = \hat{p}$.

Next, we show that the equilibrium price in the focal model falls into the interval $[\hat{p}, v)$. First, we prove that $\Pi(\hat{p}) > \Pi(p')$ for any $p' < \hat{p}$. Without loss of generality, we use retailer R_1 for illustration. Suppose retailer R_1 decreases the price from $\hat{p}$ to p', his market size will decrease, because the expected payoff of the customers is maximized at the price $p = \hat{p}$. As a result, by decreasing price from $\hat{p}$ to p', the retailer will induce a lower demand and a strictly lower profit margin. This implies that $\Pi(\hat{p}) > \Pi(p')$. Thus, the retailer should charge a price $p \geq \hat{p}$. Next, we show that the equilibrium price cannot exceed v. If the price is greater than or equal to the product valuation, i.e., $p \geq v$, no customer can afford the product, which further implies that the demand is zero and the retailer earns zero profit. Therefore, the retailer's optimal price must be within the range of $[\hat{p}, v)$.

Finally, we show that the social welfare function is concave in price p. Similar to the proof of the average customers' surplus, we fist examine the social welfare from non-switching customers. We have $SW_1(p) = v\mu\theta(p^*) - cF^{-1}\left(\frac{p^*-c}{p^*}\right) - \frac{\mu s}{4}$ and

$$\frac{dSW_1(p)}{dp} = \frac{c^2}{p^2}\left(\frac{v-p}{p}\right)\frac{1}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)} = \frac{c}{p}\left(\frac{v-p}{p}\right)\frac{1-F\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)} > 0.$$

Since the demand, D, has an increasing failure rate, $SW_1(p)$ is increasing and concave in p. Now, we examine the social welfare function from switching customers: $SW_2 = (2-\theta(p))\left[v\mu\theta(p) - cF^{-1}\left(\frac{p-c}{p}\right)\right] - \frac{\mu s}{4} - (1-\theta(p))\frac{3\mu s}{4}$. We have

$$\begin{aligned}\frac{dSW_2(p)}{dp} &= \frac{c^2}{\mu p^3}\frac{1}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}\left\{(2-\theta^*(p))(v-p)\mu - \left(v\mu\theta^*(p) - cF^{-1}\left(\frac{p-c}{p}\right)\right) + \frac{3\mu s}{4}\right\}\\ &= \frac{c}{\mu p^2}\frac{1-F\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}\left\{(2-\theta^*(p))(v-p)\mu - \left(v\mu\theta^*(p) - cF^{-1}\left(\frac{p-c}{p}\right)\right) + \frac{3\mu s}{4}\right\}.\end{aligned}$$

Note that the term in the bracket, $(2-\theta^*(p))(v-p)\mu - \left(v\mu\theta^*(p) - cF^{-1}\left(\frac{p-c}{p}\right)\right) + \frac{3\mu s}{4}$, is decreasing in price p. Since the term $\frac{1-F\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}$ is decreasing in p due to the increasing failure rate of the demand, the social welfare function is concave in p. Finally, since the total social welfare function is a weighted summation of the social welfare functions from the two customer segments, the total social welfare function $SW(p)$ is concave in price p. $\square$

Proof of Proposition 8

We first compare the two social welfare functions. Clearly, we have $p^* \le p_b^*$ because p_b^* is the maximum price that allows full market coverage and customer switching. Recall that the average consumer surplus function is decreasing in price $p \in [\hat{p}, v)$ and $p^* \in [\hat{p}, v)$, thus we have $CS^* \ge CS_b^*$.

Now, we compare the two social welfare functions. We start by examining the social welfare when the market has non-switching customers only (i.e., $\gamma = 0$). According to the proof of lemma 2, we have

$$\frac{dSW(p)}{dp} = \frac{c^2}{p^2}\left(\frac{v-p}{p}\right)\frac{1}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}.$$

Clearly, we have $\frac{dSW(p)}{dp} > 0$. Therefore, the social welfare function is strictly increasing in p. Recall that we have $p^* \le p_d^*$, $SW^* \le SW_b^*$ when $\gamma = 0$.

Next, we examine the case when the market has switching customers only (i.e., $\gamma = 1$). As shown in the proof of Proposition 5, the switching customers will always visit the retailer with lower price first, so the retailer who charges a lower price attract more demand. As a result, the retailers compete on offering lower prices when $\gamma = 1$. In equilibrium, we have $p^* = c$, so the retailers stock zero inventory and thus we have $SW^* = 0$. In contrast, the social welfare in the benchmark model is $SW_b^* > 0$ as $\theta(p_b^*) > 0$. Hence, we have $SW^* \le SW_b^*$ when $\gamma = 1$.

Finally, when the market consists of both customer types, the equilibrium price will be lower than the equilibrium price when $\gamma = 0$ and higher than that when $\gamma = 1$. Since we have proved that the market competition will lead to a lower social welfare when the market has non-switching customers and switching customers, respectively, we have $SW^* \le SW_b^*$. □

Proof of Proposition 9

First, we show $SC_c^* \ge SC^*$. Suppose the market follows the equilibrium path of the model without monetary compensation and achieves equilibrium solutions (p^*, q^*). In this case, the monetary compensation $m^* = 0$. Now, we allow the retailers to pay compensation to consumers. Accordingly, the equilibrium compensation switches from $m^* = 0$ to $m_c^* \ge 0$. A higher compensation rate increases consumer surplus and thus helps retailers earn more market share (but decreases its marginal revenue). If $m_c^* = 0$, the retailers have no incentives to compete more in market share, so the two models result in the same consumer surplus. If $m_c^* > 0$, the two retailers have incentives to compete more in market share, so a positive compensation rate raises consumer surplus. In short, since we always have $m_c^* \ge m^* = 0$, offering non-negative monetary compensation to customers upon stock can always increases the equilibrium average customer surplus, i.e., $SC_c^* \ge SC^*$.

Next, we show $SC_v^* \ge SC^*$. Similarly, suppose the market follows the equilibrium path of the base model and achieves equilibrium solution (p^*, q^*). Now, we allow the retailers to announce quantity information to the market. As a result, the market switches to a new equilibrium path (p_v^*, q_v^*) under inventory commitment. In the case of inventory commitment, the retailers are motivated to increase quantity and decrease price.

First, the retailers have incentives to increase quantity. Assume the equilibrium price p^* is unchanged. Once the retailers commit inventory to the market, the inventory quantity must not decrease. The argument is as follows. On one side, decreasing quantity decreases consumer surplus and thus decreases market share.

One the other side, deviating from the critical fractile quantity ($q = \alpha F^{-1}(\frac{p-c}{p})$) decreases marginal revenue. As a result, by decreasing inventory quantity, the retailers must earn less profit, so the inventory quantity must not be decreased. However, the retailers may increase quantity. Although increasing stock quantity also deviates from the critical fractile quantity and thus decreases the marginal revenue, it raises market share by offering higher product availability. Thus, the retailers may earn a higher profit by increasing stock quantity. Therefore, given the equilibrium price, the retailers may choose to increase quantity.

Second, the retailers have incentives to decrease price. Similarly, assume the equilibrium quantity q^* is unchanged, the retailers have no incentives to increase retail price. The argument is as follows. On one side, increasing price decreases consumer surplus and thus decreases market share. One the other side, deviating from the critical fractile price ($p = c/(1-F(\frac{q}{\alpha}))$) decreases marginal revenue. As a result, by increasing price, the retailers must earn less profit, so the retail price must not be increased. However, retailers may decrease price. Although decreasing price also deviates from the critical fractile price and thus reduces the marginal revenue, it raises market share. In other words, the retailers may earn a higher profit by decreasing price. Therefore, given the equilibrium quantity, the retailers may choose to decrease price.

In sum, once the retailers adopt the inventory commitment strategy, we have $q_v^* \geq q^*$ and(or) $p_v^* \leq p^*$. Since increasing quantity and decreasing price are both beneficial to the consumers' surplus, we have $SC_v^* \geq SC^*$. $\square$

Proof of Proposition 10

We focus on analyzing two extreme cases.

Case I. A sufficiently small search cost (i.e., $s \downarrow 0$). In this case, the entire market is fully covered with customer switching. As shown in the proof of Proposition 5, the equilibrium profit and quantity are positive in the focal model, so $SW^* > 0$. However, the retailer's profit under the inventory commitment and monetary compensation strategies are close to zero as $s \downarrow 0$, so the retailer will stock zero quantity, provide zero product availability, which leads to zero social welfare. Therefore, we have $SW_v^* < SW^*$ and $SW_c^* < SW^*$.

Case II. A large search cost. In this case, the entire market has no competition (i.e., the customer at the border has a surplus exactly equal to 0 to patronage the focal retailer) and customer switching. Moreover, each retailer can be viewed as a monopolist that serves a separate market (with a market size $\alpha < \frac{1}{2}$). Su and Zhang (2008) have proved that the inventory commitment and monetary compensation strategies provide a higher order quantity in the monopoly setting, so we have $\theta^*(p_v^*) > \theta^*(p^*)$ and $\theta^*(p_c^*) > \theta^*(p^*)$. Recall the social fare function is increasing in product availability without customer switching, so $SW_v^* > SW^*$ and $SW_c^* > SW^*$.

Finally, recall that the social welfare functions are continuous in equilibrium price p^* and the equilibrium price p^* is continuous in s, we conclude that: (1) there exists a threshold s_{vw} and we have $SW_v^* < SW^*$ if $s < s_{vw}$; and (2) there exists a threshold s_{cw} and we have $SW_c^* < SW^*$ if $s < s_{cw}$. $\square$