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Dynamic Supply Risk Management with Signal-Based Forecast, Multi-Sourcing, and Discretionary Selling

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We examine the critical role of *advance supply signals*—such as suppliers' financial health and production viability—in dynamic supply risk management. The firm operates an inventory system with multiple demand classes and multiple suppliers. The sales are *discretionary* and the suppliers are susceptible to both systematic and operational risks. We develop a hierarchical Markov model that captures the essential features of advance supply signals, and integrate it with procurement and selling decisions. We characterize the optimal procurement and selling policy, and the strategic relationship between signal-based forecast, multi-sourcing, and discretionary selling. We show that higher demand heterogeneity may *reduce* the value of discretionary selling, and that the mean value-based forecast may *outperform* the stationary distribution-based forecast. This work advances our understanding on *when* and *how* to use advance supply signals in dynamic risk management. Future supply risk erodes profitability but enhances the marginal value of current inventory. A signal of future supply shortage raises both base stock and demand rationing levels, thereby boosting the current production and tightening the current sales. Signal-based dynamic forecast effectively guides the firm's procurement and selling decisions. Its value critically depends on supply volatility and scarcity. Ignoring advance supply signals can result in misleading recommendations and severe losses. Signal-based dynamic supply forecast should be used *when*: (a) supply uncertainty is substantial, (b) supply-demand ratio is moderate, (c) forecast precision is high, and (d) supplier heterogeneity is high.

Key words: supply risk management; signal-based supply forecast; multi-sourcing; discretionary selling
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1. Introduction

To better manage supply risks, many companies closely monitor the status of their suppliers, including production schedules (Bresnahan and Ramey 1994), production progressions (Higle and Kempf 2011), preorder commitments (Aviv 2007), financial health (Babich 2010), and severe weather (Cachon et al. 2012), etc. These *advance supply signals*, though noisy and volatile, contain rich information on suppliers' likelihood of operational disruptions, bankruptcy, and, thus, their availability. If properly maintained and utilized, advance supply signals can greatly improve companies' capabilities in supply forecast and risk mitigation.

Several risk mitigation strategies are widely used, such as multi-sourcing, inter-temporal substitution, and discretionary selling. Multi-sourcing reduces capacity limitation by splitting orders across multiple suppliers. Inter-temporal substitution over orders and holds inventory in anticipation of future supply scarcity. Discretionary selling exploits customer heterogeneity over time by committing on-hand inventory to future high-value demand and rejecting current low-value orders.

Toyota, for example, takes an integrated approach of these strategies. It has long adopted the *multi-sourcing* strategy, purchasing all components from at least two suppliers (Federgruen and Yang 2011, Treece and Rechtin 1997). For example, it buys engine drive belts from suppliers such as Gates, ContiTech AG, AC Delco, and mass airflow sensors from Hitachi Automotive Systems and Denso Corporation (Matsuo 2015). In addition, Toyota uses elaborate systems to *monitor* its suppliers at all times (Liker and Choi 2004). For example, in response to the 2011 Japan earthquake, Toyota established a company-wide emergency task force to monitor its suppliers' status in real time (Jones 2012). To mitigate part shortage caused by this earthquake, Toyota leveraged multi-sourcing and ramped up production of unaffected plants. Meanwhile, it also invoked the *discretionary selling* strategy: Out of over 300,000 active part numbers, it restricted the allocation of 233 part numbers, including drive belts and mass airflow sensors, to fulfil only emergency repair orders; see, for example, Capps (2011), Ramsey and Moffett (2011). Such an integrated approach helped Toyota to navigate through the difficult times.

This study has three objectives: (1) to develop a general model for advance supply signals and signal-based supply forecast; (2) to identify the optimal procurement and selling policies under signal-based supply forecast, and (3) to characterize the value of advance supply signals. Specifically, we consider a joint procurement and selling problem under dynamic signal-based capacity forecast. The firm sources a single product from multiple suppliers and sells to multiple classes of customers. The suppliers differ in cost, capacity limit and reliability, and the customer classes in revenue. The firm tracks advance supply signals in each period, updates the capacity forecast, and makes procurement and selling decisions. The firm may deliberately overorder inventory or ration low-value demands in anticipation of future supply shortages. The firm's objective is to maximize the total expected profit over the planning horizon.

We make three contributions in this study. First, we develop a hierarchical Markov model for the signal-based dynamic supply forecast. The firm forecasts the future capacity of each supplier based on its corresponding advance supply signal. Moreover, the advance supply signal for each supplier is driven by an exogenous Markov chain. This model captures both the systematic risk and the operational risk of supply uncertainty, and is compatible with the common estimation methods in the literature.

Second, we characterize the optimal policy. The optimal procurement and selling policy has a nested threshold structure, which is specified by a sequence of supplier- and demand class-dependent monotone thresholds. The optimal procurement is driven by multi-sourcing and inter-temporal substitution; and the optimal selling is driven by customer segmentation and inter-temporal rationing. The procurement and selling decisions should be synchronized with dynamic supply forecast for adaptive and resilient risk mitigation. We also study the strategic relationships between the three risk-mitigation instruments. We find that the base stock levels with discretionary selling are lower than those without. Hence, discretionary selling and multi-sourcing are strategic substitutes. The strategic relationship between signal-based supply forecast and multi-sourcing, however, may be either complementary, substitutive, or independent, depending on the supply-demand ratio. Our analysis also reveals several counterintuitive insights on dynamic supply risk management: For example, the value of discretionary selling may be decreasing in demand heterogeneity; and the inventory system with mean capacity-based forecast may outperform that with stationary-capacity-distribution-based forecast.

Third, we characterize the effects of signal-based supply forecast in dynamic supply risk management. Signal-based dynamic supply forecast influences both procurement and selling strategies. The mechanism works as follows. Supply shortage erodes total expected profit but increases the marginal value of inventory. Hence, a signal of *future* capacity shortage increases the marginal value of *current* inventory, which in turn raises the base stock and demand rationing levels, thereby boosting production and tightening sales at the same time. Signal-based supply forecast thus plays a central role in guiding the strategic planning and coordinating the procurement and selling executions in response to evolving forecasts. The effect on profitability is immediate: Signal-based dynamic forecast enables the firm to responsively adjust its procurement and selling policies so as to better mitigate supply risks. Managerially, we show that signal-based dynamic forecast should be used when: (a) supply uncertainty is substantial, (b) supply-demand ratio is moderate, (c) forecast precision is high, and (d) supplier heterogeneity is high. We demonstrate that the conventional mean capacity-based forecast and stationary-capacity-distribution-based forecast are inadequate for handling volatile supply. Moreover, ignoring the volatility of supply signal evolution may generate misleading policies (i.e., the "average" versions of the optimal policy) and inflict severe losses.

In short, our work highlights the importance of tracking advance supply signals and explicitly modeling supply volatility for the dynamic joint procurement and selling problems with substantial supply risks. To our best knowledge, this is one of the first few attempts to understand the critical role of advance supply signals and discretionary selling in the supply risk management literature.

2. Literature Review

Our work contributes to the literature on dynamic forecast and supply risk management. Dynamic

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forecast is modeled in two ways in the existing literature. The first uses time series to capture the forecast evolution (e.g., Altug and Muharremoglu 2011, Chen and Lee 2009, Graves et al. 1998, Milner and Kouvelis 2005, Ozer and Wei 2004, Toktay and Wein 2001); the second approach employs Markov modulated demand models (e.g., Chen and Song 2001, Gao et al. 2012, Li and Gao 2008, Song and Zipkin 1993). This literature mainly focuses on how to use simple forecast models to *improve replenishment*. In contrast, we build a hierarchical Markov model to capture both the volatility and variability of advance supply signals, and use it to *coordinate* both replenishment and selling decisions for supply risk mitigation.

We also contribute to the dynamic supply risk management literature (see, e.g., Babich 2010, Federgruen, and Yang 2011, 2014, Gao 2015). By directly capturing the dynamic features of progressive information revelation and sequential decision making, our model allows for inter-temporal substitution and rationing, two commonly used strategies in dynamic supply risk management. In the supply risk management literature, the underlying supply risk is usually modeled as (a) random yield (e.g., Federgruen and Yang 2009, Henig and Gerchak 1990), (b) uncertain capacity (e.g., Chao et al. 2008, Li et al. 2013), or (c) all-or-nothing supply disruption (e.g., Gümüs et al. 2012, Tomlin 2006, Yang et al. 2009). When supply risk takes the form of procurement cost volatility, Xiao et al. (2015) study how a firm adjusts its pricing and sourcing strategies to mitigate the procurement cost fluctuation risk. See Kouvelis et al. (2011) for a comprehensive review of the supply risk management literature. In particular, uncertain capacity can realize either before (e.g., Chao et al. 2008) or after procurement decisions (e.g., Li et al. 2013). We take the first approach for technical tractability. This approach is also common in the supply disruption literature, where the supplier status (up or down) is observed at the beginning of each period. We make two contributions to this literature: (a) We incorporate advance supply signals to enhance operational instruments for better risk mitigation; (b) We analyze the interplay of forecasting, procurement, and selling, and identify the coordination role of signal-based forecast in supply risk management.

3. Signal-Based Dynamic Supply Forecast Model

In this section, we develop a hierarchical Markov model for signal-based dynamic supply forecast. Consider a firm that replenishes from a set of m suppliers $\mathcal{M}$, over a T-period planning horizon $\mathcal{T}$. Time is labeled backwards. The capacity of supplier i in period t, $K_t^i \geq 0$, is *ex ante* stochastic. Let $\mathcal{K}_t^i$ be the random variable that represents the *ex ante* distribution of K_t^i. Hence, *ex ante*, $\{\mathcal{K}_t^i\}_{i,t}$ is the underlying supply process. Without additional information, it is the best forecast for the firm. At the beginning of each period t, the firm observes the realized capacity vector $K_t = (K_t^1, \cdots, K_t^m)$.

The firm can obtain advance supply signals to *improve* its forecast. Let θ_t^i be the advance supply signal from supplier i in period t. Upon observing signal θ_t^i, the firm can update the forecast of K_{t-1}^i from its *ex ante* distribution $\mathcal{K}_{t-1}^i$ to the *ex post* signal-based one $K_{t-1}^i(\theta_t^i) = [\mathcal{K}_{t-1}^i|\theta_t^i]$. Here, $K_{t-1}^i(\theta_t^i)$ is a non-negative random variable, and the forecast function $K_{t-1}^i(\cdot)$ is stochastically increasing in θ_t^i; that is, $\mathbb{P}\{K_{t-1}^i(\hat{\theta}_t^i) > C\} \geq \mathbb{P}\{K_{t-1}^i(\theta_t^i) > C\}$ for all C and $\hat{\theta}_t^i > \theta_t^i$. The signal θ_t^i is a sufficient statistics for K_{t-1}^i: Conditioned on θ_t^i, K_{t-1}^i is independent of θ_t^j for all $j \neq i$. Moreover, the signals $\{\theta_t^i\}_t$ evolve according to an exogenous Markov process, $\theta_{t-1}^i = \Theta_{t-1}^i(\theta_t^i)$, where $\Theta_{t-1}^i(\cdot)$ is a non-negative random variable and stochastically increasing in θ_t^i. We assume that θ_t^i is a sufficient statistic for θ_{t-1}^i. Taken together, $\{K_t^i(\cdot)\}_{i,t}$ and $\{\Theta_t^i(\cdot)\}_{i,t}$ define a signal-base forecast. In our model, the randomness in capacity $\mathcal{K}_{t-1}^i$ is *resolved sequentially*: In period t, the firm observes the signal θ_t^i and updates its forecast from $\mathcal{K}_{t-1}^i$ to $K_{t-1}^i(\theta_t^i)$; in period $t-1$, as the firm observes the realization of K_{t-1}^i, its remaining randomness is completely resolved.

To characterize the (relative) forecast precision of the signal-based forecast $\{K_t(\cdot)\}_t$, we use the concept of convex order (Shaked and Shanthikumar 2007). For two random variables ϵ and $\tilde{\epsilon}$, we say ε is less than $\tilde{\epsilon}$ in *convex order* (denoted as $\epsilon \leq_{cx} \tilde{\epsilon}$), if $\mathbb{E}f(\epsilon) \leq \mathbb{E}f(\tilde{\epsilon})$ for all convex functions $f(\cdot)$. This implies $\mathbb{E}\epsilon = \mathbb{E}\tilde{\epsilon}$, and $\mathrm{var}\epsilon \leq \mathrm{var}\,\tilde{\epsilon}$, that is, ϵ is less variable than $\tilde{\epsilon}$. Let $\theta_t = (\theta_t^1, \cdots, \theta_t^m)$ be the signal vector. For two signal processes $\{\theta_t\}_t$ and $\{\tilde{\theta}_t\}_t$ and their associated forecast functions $\{K_t(\cdot)\}_t$ and $\{\tilde{K}_t(\cdot)\}_t$, we say forecast $\{K_t(\cdot)\}_t$ is more *precise* than $\{\tilde{K}_t(\cdot)\}_t$, if $K_{t-1}^i(\theta_t^i) \leq_{cx} \tilde{K}_{t-1}^i(\theta_t^i)$, $\forall i$, t, and θ_t^i. For example, if the signal θ_t is collected *after* $\tilde{\theta}_t$, then θ_t is likely to be more *informative* than $\tilde{\theta}_t$, and hence produces more precise forecast $K_{t-1}(\theta_t)$ of $\mathcal{K}_{t-1}$. Although $\{\theta_t\}_t$ and $\{\tilde{\theta}_t\}_t$ may produce capacity forecasts with different precisions, they both forecast $\{K_t\}_t$ *accurately*. This is because, *ex ante*, the forecasted capacity processes $\{K_t(\theta_t)\}_t$ and $\{\tilde{K}_t(\tilde{\theta}_t)\}_t$—based on the signal processes $\{\theta_t\}_t$ and $\{\tilde{\theta}_t\}_t$, respectively—both follow the same original distribution of $\{\mathcal{K}_t\}_t$. In section 5.3, we will analyze the impact of forecast precision under signal-based forecast.

We now illustrate our signal-based supply forecast model with a simple example. Assume that the capacity process $\{\mathcal{K}_t\}_t$ is stationary and normally distributed, with $\mathcal{K}_t^i \sim \mathcal{N}(\mu, \sigma^2)$, where μ is significantly larger than σ to ensure that the probability of K_t^i being negative is negligible. Moreover, the components of $\mathcal{K}_t$ are independent of each other. Assume that $\mathcal{K}_{t-1}^i = \theta_t^i + \xi_t^i$, where the white noise ξ_t^i is independent of signal θ_t^i and $\mathbb{E}[\xi_t^i] = 0$. The advance supply signal of supplier i, θ_t^i, follows an autoregressive process of order 1 (i.e., the $AR(1)$ process) with dynamics: $\theta_t^i = (1-\rho)\mu + \rho\theta_{t+1}^i + \sqrt{1-\rho^2}\epsilon_{t+1}^i$, where $\{\epsilon_t^i\}_t$ follow *i.i.d.* normal distribution $\mathcal{N}(0, \eta\sigma^2)$ independent of $\{\theta_t^i\}_t$, $\theta_{T+1}^i \sim \mathcal{N}(\mu, \eta\sigma^2)$, $\rho \in [0, 1]$, and $\eta \in (0, 1)$. Hence, the signals $\{\theta_t^i\}_t$ are stationary and follow the stationary distribution $\mathcal{N}(\mu, \eta\sigma^2)$. Moreover, given the advance supply signal in period t, θ_t^i, the signal-based forecast of $\mathcal{K}_{t-1}^i$ is $K_{t-1}^i(\theta_t^i) = [\mathcal{K}_{t-1}^i|\theta_t^i] \sim \mathcal{N}(\theta_t^i, (1-\eta)\sigma^2)$. Therefore, observing the advance supply signal θ_t^i in period t improves the forecast of $\mathcal{K}_{t-1}^i$ by reducing its variance from σ^2 to $(1-\eta)\sigma^2 \in (0, \sigma^2)$, which will be reduced to 0 at the beginning of period $t-1$, as the firm observes the realization K_{t-1}^i. Moreover, if η increases, $K_{t-1}^i(\theta_t^i)$ becomes smaller in the convex order and the forecast variance in period t, $(1-\eta)\sigma^2$, decreases. Thus, the forecast precision improves with a higher η value in this example.

Our model captures two types of supply risks: (1) the *systematic risk* of supply volatility, driven by significant, inter-temporal shifts of the suppliers' status (e.g., bankruptcy and natural disaster), and (2) the *operational risk* of production variability, driven by inconsequential, within-period operational variations (e.g., defects and machine breakdowns). In our model, the Markov chain $\Theta_{t-1}^i(\theta_t^i)$ captures the volatility of signal evolution – the systematic risk; and the variance of random capacity $K_{t-1}^i(\theta_t^i)$ captures the residual variability (after observing the signal θ_t^i) – the operational risk.

There is extensive empirical literature on signal-based forecast. For example, Cachon et al. (2012) establish the effects of severe weather on production; Bresnahan and Ramey (1994) calibrate the effects of production scheduling (e.g., plant shutdown, model changeover, and overtime). The estimation of specific signals is also well documented. For example, to estimate production yield, small pilot runs (Grosfeld-Nir and Gerchak 2004), inspection and on-site auditing (AppleInsider.com), and the failure mode and effect analysis (Chao et al. 2009) are widely used. Empirical estimations of Markov matrix for the evolution of advance supply signals include Bresnahan and Ramey (1994) on 8-state production scheduling in auto assembly plants, Higle and Kempf (2011) on multi-stage production throughput in semiconductor manufacturing at Intel, and Graddy and Hall (2011) on two-state weather forecast for fish supply in the Fulton Fish Market.

4. Integrated Procurement and Selling Model with Advance Supply Signals

In this section, we first formulate the decision problem as a dynamic program (section 4.1), and then decompose the optimization problem in each period into a two-stage stochastic program (section 4.2). The extensions of this base model are discussed in section 8. For vectors $x, y \in \mathbb{R}^n$, let $[x]_s^t := \sum_{i=s}^t x^i$, $|x| := \sum_{i=1}^n x^i$. We say $x \le y$ if $x^i \le y^i$ for all $i = 1, \ldots, n$. Let $\mathbf{1}_{\mathcal{A}}$ be the indicator function of event $\mathcal{A}$. Following the convention of the dynamic programming literature (Bertsekas 1995), we use the same letter (e.g., K) for a random variable and its realization. We summarize the notations in Appendix S1 (Table 2).

4.1. Formulation

Consider a single-product, periodic-review inventory system where the firm sources from a set of m suppliers, $\mathcal{M}$, to serve a set of n classes of customers, $\mathcal{N}$. In period t, each supplier $i \in \mathcal{M}$ is characterized by its unit purchasing cost c_t^i, realized capacity K_t^i, and advance supply signal θ_t^i. Let $c_t := (c_t^1, \cdots, c_t^m)$. Each demand class $j \in \mathcal{N}$ is characterized by its unit price $\tilde{r}_t^j$, and unit rejection cost b_t^j for the loss of goodwill. We will discuss the convex rejection cost case in section 8. Let $r_t^j := \tilde{r}_t^j + b_t^j$ be the effective marginal revenue. The class j demand D_t^j is continuously distributed with finite mean. Let $r_t := (r_t^1, \cdots, r_t^n)$ and $D_t := (D_t^1, \cdots, D_t^n)$. $\{D_t\}_t$ are *i.i.d.* random vectors, but the components of D_t can be correlated. In period t, we rank the suppliers by their unit purchasing cost such that $c_t^1 < \cdots < c_t^m$, and the demand classes by their effective marginal revenue such that $r_t^1 > \cdots > r_t^n$. We assume that both rankings are time invariant, but all of our results still hold when the rankings change over time, as long as we re-rank the suppliers and demand classes in each period so as to keep the suppliers' unit purchasing costs in ascending order and the demand classes' effective marginal revenues in descending order. A unit demand requires one unit on-hand inventory to fill. The initial state in period t is described by (I_t, K_t, θ_t), which represents the inventory level before replenishment I_t, the realized supply capacity K_t, and the advance supply signal θ_t. The firm's objective is to maximize its expected total profit over the planning horizon.

The sequence of events is as follows. At the beginning of period t, after observing state (I_t, K_t, θ_t), the firm makes the procurement decision $x_t = (x_t^1, \cdots, x_t^m)$ and incurs a purchasing cost $c_t \cdot x_t = \sum_{i=1}^m c_t^i x_t^i$, where $x_t^i \le K_t^i$ is the order quantity from supplier i. The suppliers deliver x_t to bring the firm's post-delivery inventory level up to $J_t = I_t + |x_t|$, where $|x_t| = \sum_{i=1}^m x_t^i$. The random demands D_t then realize. After that, the firm makes the selling decision $y_t = (y_t^1, \cdots, y_t^n)$, where $y_t^j \le D_t^j$ is the acceptance from class j, rejects the rest of the demand $D_t - y_t$ at the linear cost $\sum_{j=1}^n b_t^j(D_t^j - y_t^j)$, and collects sales revenue $\tilde{r}_t \cdot y_t = \sum_{j=1}^n \tilde{r}_t^j y_t^j$. The accepted orders y_t are filled as many as possible by the on-hand inventory; all accepted but unfilled orders are fully backlogged. Any on-hand inventory or backlog is carried over to the next period with convex inventory holding and backlogging cost $h_t(\cdot)$, with $h_t(0) = 0$. For expositional ease, we assume $h_t(\cdot)$ is continuously differentiable and $\partial h_t(\cdot)$ is uniformly bounded. At the end of period t, the firm updates the signal-based forecast by $\theta_{t-1} = \Theta_{t-1}(\theta_t)$ and $K_{t-1} = K_{t-1}(\theta_t)$. For tractability, we focus on the supplier- and demand class-independent inventory cost; we will discuss the case with supplier- and demand class-dependent inventory cost in section 8.

We formulate the planning problem as a dynamic program with discount factor $\gamma \in (0, 1)$. Let $V_t(I_t, K_t, \theta_t)$ be the maximum expected total discounted profit in periods $t, t-1, \cdots, 1$, under state (I_t, K_t, θ_t). It satisfies

$$V_t(I_t,K_t,\theta_t) = \max_{0\le x_t\le K_t}\Bigg\{-\sum_{i=1}^m c_t^i x_t^i + \mathbb{E}_{D_t}\Bigg[\max_{0\le y_t\le D_t}\begin{pmatrix}\sum_{j=1}^n \tilde{r}_t^j y_t^j - \sum_{j=1}^n b_t^j(D_t^j - y_t^j) - h_t(I_{t-1})\\ +\gamma\mathbb{E}_{(K_{t-1},\theta_{t-1})}[V_{t-1}(I_{t-1},K_{t-1},\theta_{t-1})|\theta_t],\end{pmatrix}\Bigg]\Bigg\},\quad \forall t\in\mathcal{T}, \tag{1}$$

where $I_{t-1} = I_t + |x_t| - |y_t|$, and the terminal condition is $V_0(\cdot,\cdot,\cdot) = 0$. In this formulation, $V_t(\cdot,\cdot,\cdot)$ is the maximal profit the firm can achieve under the signal-based forecast $\{K_{t-1}(\theta_t)\}_t$. Without advance signals $\{\theta_t\}_t$, the firm can only forecast with the *ex ante* distributions $\{\mathcal{K}_t\}_t$ and his profit may suffer. We compare these two scenarios analytically in section 5.1, and numerically in section 7.2.

4.2. Two-Stage Stochastic Programming Procedure

To facilitate our analysis, we first rewrite the original optimization problem in each period t as a two-stage stochastic program:

$$V_t(I_t,K_t,\theta_t) = \max\{H_t(I_t,x_t,\theta_t) : 0\le x_t\le K_t\}, \tag{2}$$

Inventory Procurement Problem

$$H_t(I_t,x_t,\theta_t) = -c_t\cdot x_t + \mathbb{E}_{D_t}\{W_t(I_t+|x_t|,D_t,\theta_t)\}, \tag{3}$$

$$W_t(J_t,D_t,\theta_t) = \max\{G_t(J_t,y_t,\theta_t) - b_t\cdot D_t : 0\le y_t\le D_t\}, \tag{4}$$

Discretionary Selling Problem

$$G_t(J_t,y_t,\theta_t) = r_t\cdot y_t - h_t(J_t-|y_t|) + \gamma\mathbb{E}_{(K_{t-1},\theta_{t-1})}[V_{t-1}(J_t-|y_t|,K_{t-1},\theta_{t-1})|\theta_t]. \tag{5}$$

We now summarize the concavity and differentiability properties of the above functions.

THEOREM 1.

(*i*) *For any t and θ_t, $G_t(J_t, y_t, \theta_t)$ is jointly concave and continuously differentiable in (J_t, y_t), $W_t(J_t, D_t, \theta_t)$ is jointly concave and continuously differentiable in (J_t, D_t), and $H_t(I_t, x_t, \theta_t)$ is jointly concave and continuously differentiable in (I_t, x_t).*

(*ii*) *For any t, $V_t(I_t, K_t, \theta_t)$ is jointly concave and continuously differentiable in (I_t, K_t) for any θ_t, and increasing in K_t^i for any (I_t, θ_t) and $i \in \mathcal{M}$.*

Let $x_t^*(I_t, K_t, \theta_t)$ and $y_t^*(J_t, D_t, \theta_t)$ be the optimal procurement and selling decisions. Although the objective functions $H_t(\cdot,\cdot,\theta_t)$ and $G_t(\cdot,\cdot,\theta_t)$ are jointly concave, the multi-dimensionality of decision variables, $(I_t, x_t) \in \mathbb{R}^{m+1}$ and $(J_t, y_t) \in \mathbb{R}^{n+1}$, still makes the conventional brute-force convex optimization procedure intractable for both analysis and computation. To overcome the dimensionality challenge, we use the problem structure to decompose the procurement problem (2) into m convex optimizations of single dimension, and the selling problem (4) in the same way. This decomposition offers a clean characterization of the optimal procurement and selling policy, reduces the computational complexity, and helps deliver insights on how the firm should coordinate the procurement and selling decisions with signal-based supply forecast. We illustrate this approach in section 4.3.

4.3. Structure of the Optimal Policy

We first characterize the structure of the optimal procurement and selling policy. Since $W_t(J_t, D_t, \theta_t)$ and, hence, $W_t(J_t, \theta_t) := \mathbb{E}_{D_t}[W_t(J_t, D_t, \theta_t)]$ are concave and continuously differentiable in J_t, we define the following threshold for each supplier i:

$$\alpha_t^i(\theta_t) := \min\{J_t\in\mathbb{R} : c_t^i \ge \partial_{J_t}W_t(J_t,\theta_t)\},\ i\in\mathcal{M}, \tag{6}$$

where $\alpha_t^i(\theta_t) := -\infty$ if $\{J_t\in\mathbb{R} : c_t^i < \partial_{J_t}W_t(J_t,\theta_t)\} = \emptyset$. Intuitively, $\alpha_t^i(\theta_t)$ is the inventory level at which the marginal value of inventory equals the marginal purchasing cost c_t^i for supplier i. Hence, $\alpha_t^i(\theta_t)$ is the base stock level if the firm orders from supplier i alone under θ_t. We now characterize the optimal procurement policy in period t.

THEOREM 2.

(*i*) *For each period t, there exists a sequence of base stock levels, $\{\alpha_t^i(\theta_t)\}_{i\in\mathcal{M}}$, that is independent of the starting inventory level I_t and capacity K_t, and is decreasing in $i \in \mathcal{M}$: $\alpha_t^1(\theta_t) \ge \alpha_t^2(\theta_t) \ge \cdots \ge \alpha_t^m(\theta_t)$, $\forall\theta_t$.*

(*ii*) *For each initial state (I_t, K_t, θ_t), there exists a supplier $i_t \in \mathcal{M}$, such that it is optimal to procure from suppliers $\{1, \cdots, i_t\}$, where*

$$i_t := m \cdot \mathbf{1}_{\{I_t+|K_t| \le \alpha_t^m(\theta_t)\}} + \min\{i \in \mathcal{M} : I_t + [K_t]_1^i \ge \alpha_t^i(\theta_t)\} \cdot \mathbf{1}_{\{I_t+|K_t| > \alpha_t^m(\theta_t)\}}.$$

(*iii*) *The optimal procurement decision is*

$$x_t^{i*}(I_t, K_t, \theta_t) = K_t^i \cdot \mathbf{1}_{\{i<i_t\}} + \min\{\alpha_t^i(\theta_t) - I_t - [K_t]_1^{i-1}, K_t^i\} \cdot \mathbf{1}_{\{i=i_t\}}, \quad i \in \mathcal{M}. \tag{7}$$

Theorem 2 shows that the optimal procurement policy is determined by the advance supply signal θ_t, the realized capacity K_t, and the starting inventory level I_t, through the optimal base stock levels $\{\alpha_t^i(\theta_t)\}_{i\in\mathcal{M}}$. It is optimal to buy from the i_t cheapest suppliers only. Moreover, the firm should order from suppliers $i < i_t$ to their full capacities K_t^i, and order $\min\{\alpha_t^{i_t}(\theta_t) - [K_t]_1^{i_t-1} - I_t, K_t^{i_t}\}$ from supplier i_t. The optimal base stock levels summarize the impact of θ_t-based forecast on the procurement policy; each $\alpha_t^i(\theta_t)$ solves a one-dimensional convex program and, hence, is computationally efficient to obtain (see Equation (6)).

The driving forces of the optimal procurement policy are: (a) current period capacity limit, which calls for multi-sourcing; and (b) inter-temporal substitution, which pools supply capacity in different decision periods in anticipation of future supply shortage. Inter-temporal substitution is implemented under the guidance of the signal-based supply forecast $\{K_{t-1}(\theta_t)\}_t$. Moreover, the θ_t-dependent base stock levels, $\{\alpha_t^i(\theta_t)\}_i$, reveal the critical role of advance supply signals in deploying inter-temporal substitution for dynamic risk management.

We now characterize the optimal selling policy. Since $V_{t-1}(I_{t-1}, K_{t-1}, \theta_{t-1})$ and, hence, $V_{t-1}(I_{t-1}|\theta_t) := \mathbb{E}_{(K_{t-1},\theta_{t-1})}[V_{t-1}(I_{t-1}, K_{t-1}, \theta_{t-1})|\theta_t]$ are concave and continuously differentiable in I_{t-1} for any θ_t, we define the following threshold for each demand class j:

$$\beta_t^j(\theta_t) := \min\{I_{t-1} \in \mathbb{R} : r_t^j \ge -\partial_{I_{t-1}} h_t(I_{t-1}) + \gamma \partial_{I_{t-1}} V_{t-1}(I_{t-1}|\theta_t)\}, j \in \mathcal{N}, \tag{8}$$

where $\beta_t^j(\theta_t) := -\infty$ if $\{I_{t-1} \in \mathbb{R} : r_t^j < -\partial_{I_{t-1}} h_t(I_{t-1}) + \gamma \partial_{I_{t-1}} V_{t-1}(I_{t-1}|\theta_t)\} = \emptyset$. Intuitively, $\beta_t^j(\theta_t)$ is the optimal demand rationing level for demand class j, that is, the desirable inventory level after satisfying demand from class j under θ_t, at which the marginal value of inventory equals the effective marginal revenue of class j.

Theorem 3.

(*i*) *There exists a sequence of rationing levels, $\{\beta_t^j(\theta_t)\}_{j\in\mathcal{N}}$, that is independent of the post-delivery inventory level J_t and realized demand D_t, and is increasing in $j \in \mathcal{N}$: $\beta_t^1(\theta_t) \le \beta_t^2(\theta_t) \le \;\; \le \beta_t^n(\theta_t), \quad \forall \theta_t$.*

(*ii*) *For each post-delivery state (J_t, D_t, θ_t), there exists a demand class $j_t \in \mathcal{N}$, such that it is optimal to satisfy the demand classes $\{1, 2, \cdots, j_t\}$, where*

$$j_t := n \cdot \mathbf{1}_{\{J_t-|D_t| \ge \beta_t^n(\theta_t)\}} + \min\{j \in \mathcal{N} : J_t - [D_t]_1^j \le \beta_t^j(\theta_t)\} \cdot \mathbf{1}_{\{J_t-|D_t| < \beta_t^n(\theta_t)\}}.$$

(*iii*) *The optimal selling decision is*

$$y_t^{j*}(J_t, D_t, \theta_t) = D_t^j \cdot \mathbf{1}_{\{j<j_t\}} + \min\{J_t - [D_t]_1^{j-1} - \beta_t^j(\theta_t), D_t^j\} \cdot \mathbf{1}_{\{j=j_t\}}, \quad j \in \mathcal{N}. \tag{9}$$

Theorem 3 shows that the optimal selling policy is determined by the advance supply signal θ_t, the realized demand D_t, and the post-delivery inventory J_t, through the optimal rationing levels $\{\beta_t^j(\theta_t)\}_{j\in\mathcal{N}}$. The firm should sell to the most profitable j_t classes only, accept all orders from classes $j < j_t$, and fulfil only $\min\{J_t - [D_t]_1^{j_t-1} - \beta_t^{j_t}(\theta_t), D_t^{j_t}\}$ from class j_t. $\{\beta_t^j(\theta_t)\}_{j\in\mathcal{N}}$ characterize the impact of signal-based supply forecast on the optimal selling policy; each $\beta_t^j(\theta_t)$ solves a one-dimensional convex program and, hence, is computationally efficient to obtain. The nested structure of the optimal selling policy relies critically on the linear rejection cost assumption. If this assumption is relaxed to a convexly increasing rejection cost, the optimal selling policy will become more involved and will not inherit the nested structure characterized in Theorem 3 (see section 8).

The optimal selling policy is driven by customer segmentation and inter-temporal rationing. Within each period, the firm should sell to the most lucrative orders through customer segmentation; across periods, it exercises inter-temporal rationing to reduce future supply-demand mismatches by reserving current inventory for future high-value demand. Thus, discretionary selling enables the firm to avoid excessive delay penalties by denying unprofitable orders upfront and, in the meantime, improve future premium customers' service (and thus profitability) through stock reservation. Next, we characterize how inventory level impacts the optimal policy.

Theorem 4.

(*i*) *The optimal procurement quantity from supplier $i \in \mathcal{M}$, $x_t^{i*}(I_t, K_t, \theta_t)$, is decreasing in the starting inventory level I_t. Moreover, for any $i \in \mathcal{M}$ and $\delta > 0$,*

$$x_t^{i*}(I_t, K_t, \theta_t) \geq x_t^{i*}(I_t + \delta, K_t, \theta_t) \geq x_t^{i*}(I_t, K_t, \theta_t) - \delta, \tag{10}$$

$$|x_t^*(I_t, K_t, \theta_t)| \geq |x_t^*(I_t + \delta, K_t, \theta_t)| \geq |x_t^*(I_t, K_t, \theta_t)| - \delta. \tag{11}$$

(*ii*) *The optimal selling quantity in class* $j \in \mathcal{N}$, $y_t^{j*}(J_t, D_t, \theta_t)$, *is increasing in the post-delivery inventory level* J_t. *Moreover, for any* $j \in \mathcal{N}$ *and* $\delta > 0$,

$$y_t^{j*}(J_t, D_t, \theta_t) \leq y_t^{j*}(J_t + \delta, D_t, \theta_t) \leq y_t^{j*}(J_t, D_t, \theta_t) + \delta, \tag{12}$$

$$|y_t^*(J_t, D_t, \theta_t)| \leq |y_t^*(J_t + \delta, D_t, \theta_t)| \leq |y_t^*(J_t, D_t, \theta_t)| + \delta. \tag{13}$$

Theorem 4 formalizes the effects of inventory on optimal procurement and selling decisions at both the individual supplier/demand class level and the aggregate level. Additional inventory reduces the need for inventory replenishment, multi-sourcing, and discretionary selling. More specifically, the aggregate procurement [sales] quantity $|x_t^*(I_t, K_t, \theta_t)|$ [$|y_t^*(I_t, D_t, \theta_t)|$] decreases [increases] at most at the same rate as the starting [post-delivery] inventory level increases.

5. Impact of Advance Supply Signals

In this section, we first demonstrate the profit improvement and policy adjustment of adopting signal-based supply forecast. Then we characterize how the firm should respond to advance supply signal volatility by dynamically adjusting the procurement and selling policies. Finally, we characterize the impact of the magnitude and precision of the forecasted capacity.

5.1. Value and Impact of Signal-Based Supply Forecast

In some scenarios, the firm could not track the advance supply signals in each period. Instead, it forecasts future supply capacities based on their *ex ante* distribution $\{\mathcal{K}_t^i\}_t$. In this subsection, for expositional ease, we assume that $\{\mathcal{K}_t^i\}_t$ follows a stationary distribution throughout the planning horizon. The supply forecast based on $\{\mathcal{K}_t^i\}_t$ for each supplier i is referred to as the *stationary forecast* of $\{K_t^i\}_t$. The results can be adapted easily for non-stationary capacity processes.

The stationary distribution $\mathcal{K}_t^i$ can be approximated by the empirical distribution of the realized capacities before the start of the planning horizon, $\{K_t^i\}_{t > T}$. We assume that the Markov chain $\{\theta_t\}_t$ is ergodic, so it has a stationary distribution and the empirical distribution of $\{\theta_t\}_{t > T}$ converges to the stationary distribution of $\{\theta_t\}_t$. Hence, $\{K_t\}_t$ also has a stationary distribution to which the empirical distribution of $\{K_t\}_{t > T}$ converges. Let $\{\mathcal{K}_t\}_t$ be the *i.i.d.* random vectors with the stationary distribution of $\{K_t\}_t$. Therefore, without signal-based forecast, the firm forecasts the next period capacities K_{t-1} with $\mathcal{K}_{t-1}$.

We now investigate the policy implications of using the stationary forecast, when the underlying capacity evolves as $\{\mathcal{K}_t\}_t$. We use the notation "ˆ" to denote the inventory system under the stationary forecast $\{\mathcal{K}_t\}_t$. The planning problem under $\{\mathcal{K}_t\}_t$ can be formulated as

$$\hat{V}_t(I_t, K_t) = \max_{0 \leq x_t \leq K_t} \Bigg\{ -\sum_{i=1}^m c_t^i x_t^i + \mathbb{E}_{D_t} \left[\max_{0 \leq y_t \leq D_t} \begin{pmatrix} \sum_{j=1}^n \tilde{r}_t^j y_t^j - \sum_{j=1}^n b_t^j (D_t^j - y_t^j) \\ -h_t(I_{t-1}) + \gamma \mathbb{E}_{\mathcal{K}_{t-1}}[\hat{V}_{t-1}(I_{t-1}, \mathcal{K}_{t-1})] \end{pmatrix} \right] \Bigg\}, \quad t \in \mathcal{T}, \tag{14}$$

where $I_{t-1} = I_t + |x_t| - |y_t|$, $\hat{V}_t(\cdot, \cdot)$ is the value function, and $\hat{V}_0(\cdot, \cdot) = 0$. $\hat{V}_t(\cdot, \cdot)$ is the maximum expected profit from period t till the end of the planning horizon, when the firm only knows (and thus forecasts with) the *ex ante* distribution $\{\mathcal{K}_t\}_t$. As in section 4.3, the optimal procurement policy for Equation (14) has the nested threshold structure specified by base stock levels $\{\hat{\alpha}_t\}_t$, and the selling policy by rationing levels $\{\hat{\beta}_t\}_t$. See Theorem 9 in Appendix S3 for details. Clearly, in each period t, the control parameters $\{\hat{\alpha}_t^i\}_{i \in \mathcal{M}}$ and $\{\hat{\beta}_t^j\}_{j \in \mathcal{N}}$ under the stationary forecast are independent of the signals θ_t, which contain more precise information on the future supply condition than the stationary capacity distribution $\{\mathcal{K}_{\theta u}\}_{\theta u \leq t}$.

The optimal policy under stationary-distribution-based forecast $(\hat{x}_t^*(I_t, K_t), \hat{y}_t^*(J_t, D_t))$ does not take into account the volatility of the system captured by the advance supply signals $\{\theta_t\}_t$. Thus, signal-based dynamic supply forecast improves the firm's profit by facilitating the firm to track the capacity status of the suppliers and to respond to the capacity volatility in real time. In section 7.2, we will numerically quantify the value of advance supply signals, and further demonstrate under what conditions signal-based supply forecast is most beneficial.

We now study the policy implications of signal-based supply forecast. The next lemma establishes the relationship between the marginal value of the on-hand inventory under the signal-based forecast and that under the stationary forecast, which is useful for policy comparison.

LEMMA 1. *Assume that, for each $i \in \mathcal{M}$, $\{\theta_t^i\}$ is ergodic and has a stationary distribution. For each period t, there exist two threshold vectors $\bar{\theta}_t = (\bar{\theta}_t^1 \cdots, \bar{\theta}_t^m)$ and $\underline{\theta}_t = (\underline{\theta}_t^1 \cdots, \underline{\theta}_t^m)$, such that*

$$\begin{cases} \partial_I \mathbb{E}_{K_{t-1},\theta_{t-1}}[V_{t-1}(I, K_{t-1}, \theta_{t-1})|\theta_t] \\ \quad \leq \partial_I \mathbb{E}_{\mathcal{K}_{t-1}}[\hat{V}_{t-1}(I, \mathcal{K}_{t-1})], \quad\quad if\ \theta_t \geq \bar{\theta}_t, \\ \partial_I \mathbb{E}_{K_{t-1},\theta_{t-1}}[V_{t-1}(I, K_{t-1}, \theta_{t-1})|\theta_t] \\ \quad \geq \partial_I \mathbb{E}_{\mathcal{K}_{t-1}}[\hat{V}_{t-1}(I, \mathcal{K}_{t-1})], \quad\quad if\ \theta_t \leq \underline{\theta}_t. \end{cases} \tag{15}$$

Based on Lemma 1, we now compare the optimal policies under signal-based dynamic forecast and those under the stationary forecast without advance supply signals.

THEOREM 5. *Assume that, for each $i \in \mathcal{M}$, $\{\theta_t^i\}$ is ergodic and has a stationary distribution. For each period t, for any I_t, J_t, K_t, and D_t, $i \in \mathcal{M}$, $j \in \mathcal{N}$,*

$$\begin{aligned} &if\ \theta_t \geq \bar{\theta}_t,\ then\ \alpha_t^i(\theta_t) \leq \hat{\alpha}_t^i, x_t^{i*}(I_t, K_t, \theta_t) \leq \hat{x}_t^{i*}(I_t, K_t), \\ &\beta_t^j(\theta_t) \leq \hat{\beta}_t^j,\ and\ y_t^{j*}(J_t, D_t, \theta_t) \geq \hat{y}_t^{j*}(J_t, D_t); \end{aligned} \tag{16}$$

$$\begin{aligned} &if\ \theta_t \leq \underline{\theta}_t,\ then\ \alpha_t^i(\theta_t) \geq \hat{\alpha}_t^i, x_t^{i*}(I_t, K_t, \theta_t) \geq \hat{x}_t^{i*}(I_t, K_t), \\ &\quad \beta_t^j(\theta_t) \geq \hat{\beta}_t^j,\ and\ y_t^{j*}(J_t, D_t, \theta_t) \leq \hat{y}_t^{j*}(J_t, D_t). \end{aligned} \tag{17}$$

Theorem 5 shows that the optimal policy under the stationary forecast is an "*average*" version of that under the signal-based forecast: For each supplier and demand class, the optimal base stock and rationing levels under the stationary forecast are between those under the signal-based forecast when the firm receives sufficiently high and low signals (captured by $\bar{\theta}_t$ and $\underline{\theta}_t$, respectively). That is, for any $\tilde{\theta}_t \leq \underline{\theta}_t$ and $\hat{\theta}_t \geq \bar{\theta}_t$, we have $\hat{\alpha}_t^i \in [\alpha_t^i(\hat{\theta}_t), \alpha_t^i(\tilde{\theta}_t)]$ and $\hat{\beta}_t^j \in [\beta_t^j(\hat{\theta}_t), \beta_t^j(\tilde{\theta}_t)]$ for $i \in \mathcal{M}$ and $j \in \mathcal{N}$.

This reveals how advance supply signals improve the deployment of procurement and selling. Relative to the optimal policy without advance supply signals, $\{\hat{x}_t^{i*}\}_{i,t}$ and $\{\hat{y}_t^{j*}\}_{j,t}$, a high supply signal prompts the firm to reduce the number of active suppliers and to turn away fewer customers (see Equation (16)), whereas a low supply signal implies a larger number of active suppliers and rejecting more customers (see Equation (17)). Employing signal-based supply forecast results in more *adaptive* buying and selling decisions, better deployment of capacity and inventory, and, thereby, better supply-demand match. In contrast, the stationary forecast ignores the volatility of advance supply signals and aggregates real fluctuations of the environment. Consequently, it produces potentially misleading policies $\{\hat{x}_t^{i*}\}_{i,t}$ and $\{\hat{y}_t^{j*}\}_{j,t}$ that are not adaptive to evolving supply conditions $\{\theta_t\}_t$. As we will illustrate in section 7.2, the cost of ignoring advance supply signals $\{\theta_t\}_t$ and, thus, the systematic risk can be staggering.

5.2. Optimal Policy in Response to Advance Supply Signals

As shown in section 5.1, the firm benefits from signal-based supply forecast because it enables *adaptive response* to the evolution of advance supply signals. We now specify how the firm should carry out such an adaptive response.

THEOREM 6.

(*i*) *If $\theta_t \leq \hat{\theta}_t$, then $V_{t-1}(I_{t-1}|\theta_t) \leq V_{t-1}(I_{t-1}|\hat{\theta}_t)$ and $V_t(I_t, K_t, \theta_t) \leq V_t(I_t, K_t, \hat{\theta}_t)$.*

(*ii*) *If $\theta_t \leq \hat{\theta}_t$, then $\partial_{J_t} W_t(J_t, \theta_t) \geq \partial_{J_t} W_t(J_t, \hat{\theta}_t)$, $\partial_{I_{t-1}} V_{t-1}(I_{t-1}|\theta_t) \geq \partial_{I_{t-1}} V_{t-1}(I_{t-1}|\hat{\theta}_t)$, and $\partial_{I_t} V_t(I_t, K_t, \theta_t) \geq \partial_{I_t} V_t(I_t, K_t, \hat{\theta}_t)$.*

(*iii*) *If $\theta_t \leq \hat{\theta}_t$, then, for any $i \in \mathcal{M}$ and $j \in \mathcal{N}$, $\alpha_t^i(\theta_t) \geq \alpha_t^i(\hat{\theta}_t)$, $x_t^{i*}(I_t, K_t, \theta_t) \geq x_t^{i*}(I_t, K_t, \hat{\theta}_t)$, $\beta_t^j(\theta_t) \geq \beta_t^j(\hat{\theta}_t)$, and $y_t^{j*}(J_t, D_t, \theta_t) \leq y_t^{j*}(J_t, D_t, \hat{\theta}_t)$.*

Part (*i*) shows that higher advance supply signals imply higher future capacities and, thus, higher expected profit. Part (*ii*) reveals that a lower advance supply signal raises the marginal value of both starting and post-delivery inventories in each period t. Part (*iii*) implies that, in anticipation of future scarcity (i.e., a low θ_t), the firm should take the coordinated response of ordering more and selling less to reserve inventory for future high-value demand. Theorem 6 implies that future supply information could influence the current procurement and selling decisions. One finding in the existing disruption literature is that procurement should adapt to the future disruption probability (Li et al. 2004). We advance this finding by showing that selling should adapt to the supply information as well, and that both procurement and selling should be coordinated by advance supply signals.

5.3. Impact of Capacity Forecast under Signal-Based Supply Forecast

In this subsection, we demonstrate the impact of forecast precision and future capacity availability under the signal-based supply forecast. As discussed in section 3, the convex order well captures the forecast precision of signal-based forecasts: If $K_{t-1}^i(\theta_t^i) \leq_{cx} \tilde{K}_{t-1}^i(\theta_t^i), \forall i, t$, and θ_t^i, then the system under $\{K_t(\cdot)\}_t$ predicts supply more precisely than the system under $\{\tilde{K}_t(\cdot)\}_t$ does. Let $\leq_{s.d.}$ denotes the first order stochastic dominance. The following theorem demonstrates how the forecast precision and future capacity availability influence the system performance.

THEOREM 7.

(*i*) *If two systems are equivalent except that* $K^i_{t-1}(\theta_t) \leq_{cx} \tilde{K}^i_{t-1}(\theta_t)$, $\forall t$ *and* $i \in \mathcal{M}$, *we have* $V_t(I_t, K_t, \theta_t) \geq \tilde{V}_t(I_t, K_t, \theta_t)$ *for any t.*

(*ii*) *For each* θ_t, $V_t(I_t, K_t, \theta_t)$ *is submodular in* (I_t, K^i_t) *for all t and* $i \in \mathcal{M}$. *If two systems are equivalent except that, for every t and* $i \in \mathcal{M}$, $K^i_{t-1}(\theta_t) \leq_{s.d.} \tilde{K}^i_{t-1}(\theta_t)$, *then* $\partial_{J_t} W_t(J_t, \theta_t) \geq \partial_{J_t} \tilde{W}_t(J_t, \theta_t)$, $\partial_{I_{t-1}} V_{t-1}(I_{t-1}|\theta_t) \geq \partial_{I_{t-1}} \tilde{V}_{t-1}(I_{t-1}|\theta_t)$, *and* $\partial_{I_t} V_t(I_t, K_t, \theta_t) \geq \partial_{I_t} \tilde{V}_t(I_t, K_t, \theta_t)$.

(*iii*) *If two systems are equivalent except that* $K^i_{t-1}(\theta_t) \leq_{s.d.} \tilde{K}^i_{t-1}(\theta_t)$ *for every t and* $i \in \mathcal{M}$, *we have, for all* $i \in \mathcal{M}$ *and* $j \in \mathcal{N}$, $\alpha^i_t(\theta_t) \geq \tilde{\alpha}^i_t(\theta_t)$, $x^{i*}_t(I_t, K_t, \theta_t) \geq \tilde{x}^{i*}_t(I_t, K_t, \theta_t)$, $\beta^j_t(\theta_t) \geq \tilde{\beta}^j_t(\theta_t)$, *and* $y^{j*}_t(J_t, D_t, \theta_t) \leq \tilde{y}^{j*}_t(J_t, D_t, \theta_t)$.

Theorem 7(*i*) characterizes the impact of supply forecast precision and reveals that more precise supply forecast enhances the firm's profitability, thus highlighting the importance of improving forecast precision in risk management. Moreover, parts (*ii*) and (*iii*) reveal how the availability of future capacities influences the current procurement and selling decisions: A lower capacity forecast increases the marginal value of current stock, which in turn increases base stock and rationing levels, and, hence, expands procurement and tightens sales in the current period.

Together, Theorems 6 and 7 pin down the impact of both systematic and operational risks in dynamic supply risk management. As shown by Theorem 6, the systematic risk, captured by the *volatility* of advance supply signals $\{\theta_t\}_t$, has significant impact on the profit and policy. The firm should track the advance supply signal θ_t in real time and synchronize procurement and selling decisions in response to the stochastically evolving θ_t. Theorem 7 shows that the operational risk, captured by the *variability* of the capacity distributions $\{K^i_{t-1}(\theta_t)\}_i$, also influences the profit and the optimal policy. The firm should fine-tune its policy execution to the magnitude and precision of the forecasted capacity.

6. Impact of Discretionary Selling

Though absent from the supply risk literature, discretionary selling is widely used for the risk mitigation purpose in business practice. The effectiveness of discretionary selling lies in its ability to discriminate against lower-margin demands, reserving scarce stock for future high-margin ones. As discretionary selling allows for responsive control of sales after demand realization, the demand–supply mismatch cost can be substantially reduced. Without discretionary selling, the planning problem reduces to the multi-sourcing inventory problem with full backlogging. We use "ˇ" to denote the inventory system without discretionary selling. Hence, the planning problem for the firm without discretionary selling can be formulated as the following dynamic program.

$$\check{V}_t(I_t, K_t, \theta_t) = \max_{0 \leq x_t \leq K_t} \left\{ -\sum_i c^i_t x^i_t + \mathbb{E}_{(D_t, K_{ty-1}, \theta_{t-1})} \left[\sum_j \tilde{r}^j_t D^j_t - h_t(I_{t-1}) + \gamma \check{V}_{t-1}(I_{t-1}, K_{t-1}, \theta_{t-1}) \middle| \theta_t \right] \right\}, t \in \mathcal{T}, \tag{18}$$

where $I_{t-1} = I_t + |x_t| - |D_t|$, $\check{V}_t(\cdot, \cdot, \cdot)$ is the value function, and $\check{V}_0(\cdot, \cdot, \cdot) = 0$. The optimal procurement decision $\check{x}^i_t(I_t, K_t, \theta_t)$ is again specified by base stock levels $\{\check{\alpha}^i_t(\theta_t)\}_t$; see Theorem 10 in Appendix S3 for details. We now characterize the impact of discretionary selling on the procurement policy.

THEOREM 8.

(*i*) *For each t,* θ_t, *and* $i \in \mathcal{M}$, $\alpha^i_t(\theta_t) \leq \check{\alpha}^i_t(\theta_t)$, *and* $x^{i*}_t(I_t, K_t, \theta_t) \leq \check{x}^{i*}_t(I_t, K_t, \theta_t)$.

(*ii*) *For each t,* θ_t, $i \in \mathcal{M}$ *and* $j \in \mathcal{N}$, $\check{\alpha}^i_t(\theta_t)$ *and* $\check{x}^{i*}_t(I_t, K_t, \theta_t)$ *do not depend on* $\tilde{r}^j_s$, *for any* $s \in \mathcal{T}$.

(*iii*) *For each t,* θ_t, $i \in \mathcal{M}$ *and* $j \in \mathcal{N}$, $\alpha^i_t(\theta_t)$ *and* $x^{i*}_t(I_t, K_t, \theta_t)$ *are increasing in* $\tilde{r}^j_s$, *for any* $s \leq t$.

Theorem 8(*i*) shows that the optimal base stock levels without discretionary selling are upper bounds of their counterparts with discretionary selling. Compared with the firm that cannot discretionarily sell its product, the firm with discretionary selling may intentionally ration its demand and have lower actual demand. Hence, the firm with discretionary selling should set lower base stock levels. Another impact of discretionary selling is that it drives the firm to set base stock levels increasing in the effective marginal revenues, as shown in Theorem 8(*iii*). With higher effective marginal revenues, the firm with the discretionary selling strategy is motivated to increase its order quantities to extract higher revenues.

Intuitively, the value of discretionary selling is mainly driven by the potential to intentionally limit the demand from low-value demand classes so as to reserve inventory for future high-value demand classes. Hence, one may conjecture that the value of discretionary selling is enhanced by higher demand class heterogeneity (characterized by the ratios between the effective marginal revenues of different demand classes). However, Theorem 8(*iii*) demonstrates that if the effective marginal revenue for the high-value demand class increases (thus, the demand class heterogeneity also increases), the firm will increase its base stock levels and, hence, hold more inventory. With ample supply of on-hand inventory, the firm is less likely to ration its demand and, hence, the value of discretionary selling decreases. Therefore, to our surprise, the value of discretionary selling may increase or decrease with the demand class heterogeneity. In section 7.5, we present our numerical results on the value of discretionary selling that confirm this counterintuitive insight.

7. Numerical Studies

This section quantifies the impact of future supply uncertainty (section 7.1), the value of advance supply signals (section 7.2), the difference between forecasting with stationary capacity distribution and that with capacity mean (section 7.3), the strategic relationship between signal-based forecast and multi-sourcing (section 7.4), and the value of discretionary selling (section 7.5).

We consider three forecasting schemes {DF, SF, MF}: the *Dynamic Forecast* (DF), based on advance supply signals; the *Stationary Forecast* (SF), based on the stationary distribution of capacity; the *Mean Forecast* (MF), based on the mean of the stationary distribution. Clearly, the stationary forecast ignores the *volatility* (the inter-temporal dependence of the signal evolution), whereas the mean forecast disregards both the *volatility* and the *variability* of the uncertain future supply. Only dynamic forecast embraces both the *volatility* and *variability* information carried by the advance supply signals θ_t. We evaluate the optimal profit under each forecasting scheme and compare the optimal profits across different schemes. The optimal profit under forecasting scheme F_i is denoted as V^{F_i}. In evaluating V^{F_i}, we take $I_t = 0$ as the reference inventory level.

We specify the model parameters as follows. The base scenario considers a Markov system with $T = 5$, two suppliers with $c_1 < c_2 = 30$, two demand classes with $(r_1, r_2) = (50, 48)$, $(b_1, b_2) = (0, 0)$, and *i.i.d.* demand streams with D_1, $D_2 \sim \mathcal{N}(6, 1^2)$. Linear unit holding and backorder costs are $h = 1$, $b = 10$. The advance supply signals $\{\theta_t^1\}_t$ and $\{\theta_t^2\}_t$ of two suppliers are independent; so are their capacity processes. For supplier i, its advance supply signals $\theta_t^i \in \{1, 2\}$ evolve as a Markov chain with transition matrix $\begin{bmatrix} p_\theta & 1-p_\theta 1 \\ -p_\theta & p_\theta \end{bmatrix}, p_\theta \geq 0.5$; its capacities $K_{t-1}^i \in \{K_l, K_h\}$. The signal-based supply forecast of supplier i in period t, $K_{t-1}^i(\theta_t^i)$, follows the conditional probability distributions $[P(K_{t-1}^i|\theta_t^i)] = \begin{bmatrix} p_K & 1-p_K \\ 1-p_K & p_K \end{bmatrix}$, $p_K \geq 0.5$. Direct computation yields that the stationary distribution of K_t^i is (0.5, 0.5). Hence, p_K measures the *forecast precision* with signal-based supply forecast: the higher the p_K, the more precise and informative the advance supply signals.

We generate testing scenarios by systematically varying the key parameters from the base scenario. All of our results are robust and hold for a large variety of parameter specifications. For brevity, we only report the results of the typical examples.

7.1. Impact of Future Supply Uncertainty

Future supply uncertainty arises from the operational risk (i.e., the *variability* of the capacity distribution) and the systematic risk (i.e., the *volatility* of the capacity evolution). We quantify by $\lambda(\sigma_K) = (V_{iid}^{SF} - V_{iid}^{MF})/V_{iid}^{SF}$ the loss of ignoring variability information under stationary *i.i.d.* capacities (Figure 1), and by $\lambda(\theta_t) = (V^{DF} - V^{MF})/V^{DF}$ the loss of ignoring both variability and volatility information under Markov capacities (Figure 2). The test setup is $(K_l, K_h) \in \{(12, 12), (11, 13), \ldots, (0, 24)\}$, $\mathbb{E}K_t = 12$, $p_\theta = 0.5$, $p_K = 1$, $(r_1, r_2) = (50, 48)$, $c_1 \in \{5, 10, 20\}$.

Our numerical results show that the cost of ignoring supply uncertainty is significant (up to 18%) and increasing in supply variability (captured by the coefficient of variation of the stationary supply process of each supplier). Intuitively, higher supply variability reduces the forecast precision of MF and compromises the control precision, resulting in more demand-supply mismatches and, hence, lower profits. In Figure 1, the profit loss is mainly driven by ignoring the operational risk, while in Figure 2 it is driven by ignoring both the systematic and operational risks. In our extensive tests, we find that the profit loss is substantial even when the capacity process is stationary, and will increase when the capacity distributions are time-

Figure 1 Impact of Capacity Variability: The *i.i.d.* Capacity Case

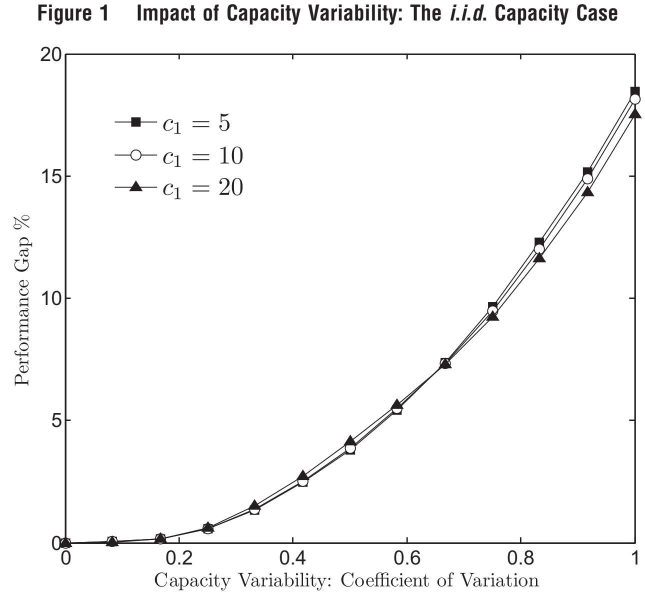

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Figure 2 Impact of Capacity Variability: The Markov Capacity Case

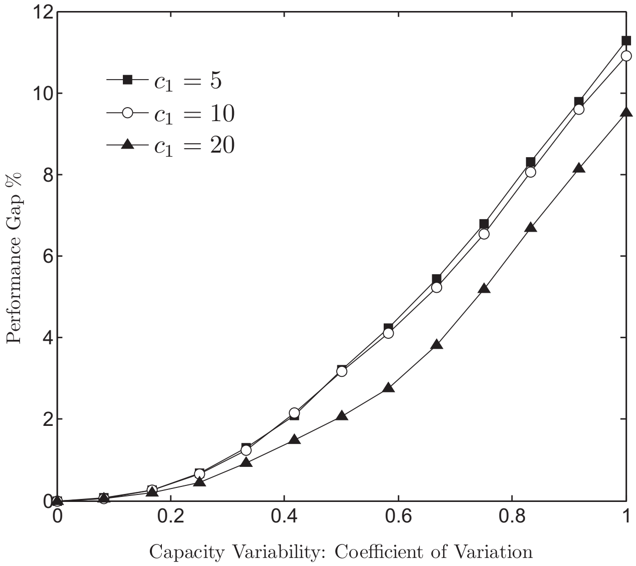
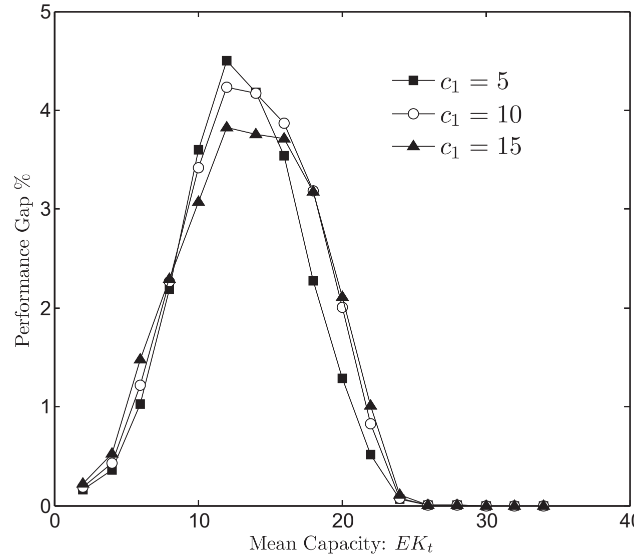


Figure 3 Impact of Mean Capacity on λ_A



varying. These results suggest that firms must explicitly account for supply variability and volatility: Conventional deterministic approximation is inadequate for turbulent environments.

7.2. Value of Advance Supply Signals

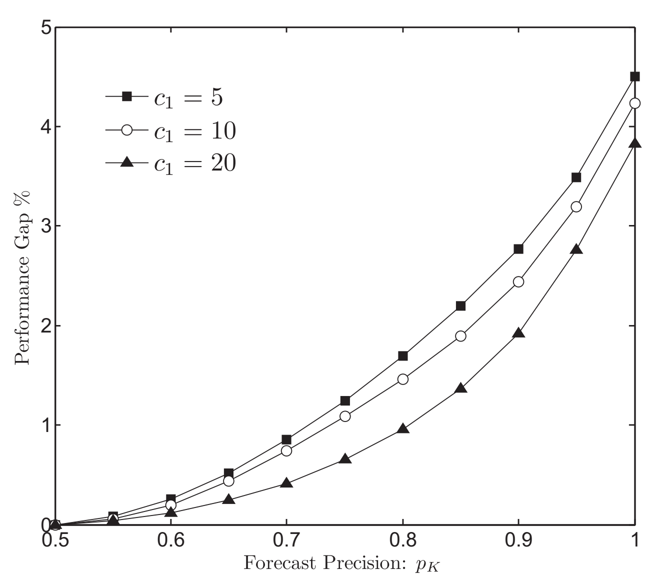
By providing early warnings of the systematic risk, advance supply signals help improve system responsiveness, resilience, and, hence, profitability. We quantify the value of advance supply signals by $\lambda_A = (V^{\mathrm{DF}} - V^{\mathrm{SF}})/V^{\mathrm{SF}}$. This value depends heavily on supply-demand ratio (Figure 3) and the forecast precision p_K (Figure 4). We design tests as $(K_l, K_h) \in \{(0, 4), (2, 6), \ldots, (22, 46)\}$ with $\mathbb{E}K_t \in \{2, 4, \ldots, 34\}$, $p_\theta = 0.5$, $p_K = 1, (r_1, r_2) = (35, 33)$.

Figure 3 shows that the benefit of signal-based forecast is most significant when the supply-demand ratio is moderate. Note that the value of advance supply signals in forecasting is primarily driven by the inter-temporal substitution and inter-temporal rationing. When the supply-demand ratio is very low, the supply scarcity is likely to occur in every period. In this case, the firm should fully utilize the capacity of all suppliers no matter what advance supply signal it receives; hence the value of signal-based supply forecast is limited. On the other hand, when the supply-demand ratio is very high, the supply scarcity is unlikely to occur. Hence, regardless of the advance supply signal, it is not optimal to use the inter-temporal substitution by over-ordering or inter-temporal rationing by rejecting low-value orders. Therefore, it is only when the supply-demand ratio is moderate that the value of signal-based supply forecast is significant. In this case, advance supply signals enable the firm to employ the inter-temporal substitution and inter-temporal rationing most effectively.

By varying p_K on the range [0.5, 1], Figure 4 shows that the value of advance supply signal increases in forecast precision p_K. The higher the p_K, the more precise the signal-based forecast, and the more valuable the advance supply signals. Therefore, to make the best use of advance supply signals, the firm should improve the signal quality.

Our results shed light on the limitation of the stationary supply forecast approach, which ignores the *volatility* of supply capacity evolution. As our numerical results demonstrate, although the stationary forecast performs well when the supply-demand ratio is consistently low or high, the resulting profit deteriorates sharply for the case with moderate supply-demand ratio, which is prevalent in practice. Indeed, this is the case that careful management and precise

Figure 4 Impact of Forecast Precision p_K on λ_A



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forecast matter most. This new insight underscores the importance of leveraging advance supply signals and explicitly modeling supply volatility: Stationary supply forecast cannot satisfactorily cope with volatile supply in general.

7.3. Stationary Forecast Vs. Mean Forecast

When advance supply signals are not available, the stationary forecast and the mean forecast are commonly used. We now quantify how capacity variability (Figure 5) and signal evolution volatility (Figure 6) affect their relative performance by the performance gap $\lambda_{\mathrm{MF}}^{\mathrm{SF}} = (V^{\mathrm{SF}} - V^{\mathrm{MF}})/V^{\mathrm{MF}}$. The test setup is $(K_l, K_h) = \{(0, 24), (2, 22), \ldots, (10, 14)\}$ with $\mathbb{E}K_t = 12, p_\theta = 0.75, p_K = 1, (r_1, r_2) = (50, 48)$.

Figure 5 shows that, in general, the stationary forecast outperforms the mean forecast and the performance gap $\lambda_{\mathrm{MF}}^{\mathrm{SF}}$ increases in capacity variability. Intuitively, though the stationary forecast dismisses signal evolution (systematic risk), it still captures supply variability (operational risk). Mean forecast, however, ignores both risks, and, hence, loses more profit in general.

Figure 6 examines the impact of the volatility of signal evolution on $\lambda_{\mathrm{MF}}^{\mathrm{SF}}$. In this numerical example, $p_\theta \in [0.5, 1]$, $(K_l, K_h) = (0, 24)$. Here, the higher the transition probability p_θ, the less volatile the signal evolution. Stationary forecast outperforms the mean forecast when evolution volatility is moderate to high ($p_\theta < 0.95$). This is intuitive, as the stationary forecast captures the operational risk (by the stationary distribution of the capacity), which is ignored by the mean forecast. Surprisingly, the mean forecast may outperform the stationary forecast ($\lambda_{\mathrm{MF}}^{\mathrm{SF}} < 0$), when evolution volatility is low ($p_\theta \uparrow 1$). In this case, the system will be trapped in the same state for an extended period of time with high probability. This behavior is better predicted by the mean forecast, rather than the *i.i.d.* dynamics projected by the stationary forecast.

Figure 5 Impact of Capacity Variability on $\lambda_{\mathrm{MF}}^{\mathrm{SF}}$

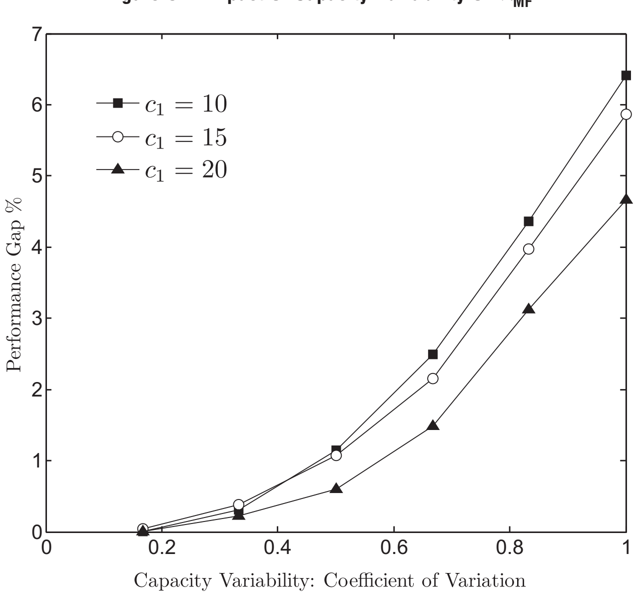
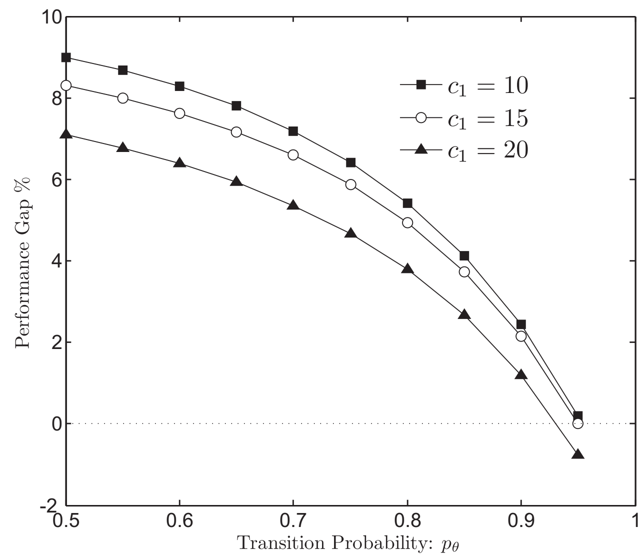


Figure 6 Impact of Evolution Volatility on $\lambda_{\mathrm{MF}}^{\mathrm{SF}}$



7.4. Strategic Relationship between Signal-Based Forecast and Multi-Sourcing

In supply risk management, signal-based dynamic forecast derives its value from the ability to calibrate decisions to actual risk levels, whereas multi-sourcing obtains its value from increasing sourcing flexibility and from reducing purchasing cost by supplier substitution.

We now quantify the relationship between these two strategies. Table 1 reports the cross difference of their profits, $\lambda_R = (V^{\mathrm{DF}} - V^{\mathrm{SF}}) - (V_1^{\mathrm{DF}} - V_1^{\mathrm{SF}})$, where V^{DF} and V^{SF} are the optimal profits with multi-sourcing under the dynamic and stationary forecasts, and V_1^{DF} and V_1^{SF} are the counterparts with single-sourcing from supplier 2. Thus, the two strategies are complementary if $\lambda_R > 0$, independent if $\lambda_R = 0$, and substitutive if $\lambda_R < 0$. We test the scenarios of $(K_l, K_h) \in \{(0, 12), (2, 14), , (18, 30)\}$ with $\mathbb{E}K_t \in \{6, 8, , 24\}$, $p_\theta = 0.75$, $p_K = 1$, $(r_1, r_2) = (50, 45)$, $c_1 \in \{20, 25, 30, 35\}$.

We find that the relationship between multi-sourcing and signal-based supply forecast depends on the supply-demand ratio and the cost structure: (a) When the supply-demand ratio is low, capacity is scarce and the two strategies are complements ($\lambda_R > 0$), because dynamic forecast can leverage the value of additional resource from multi-sourcing in combating shortage. (b) When the supply-demand ratio is moderate, capacity is ample and the two strategies are substitutes. With ample supply capacity from a single supplier, the potential benefit from the greater

Table 1 Strategic Relationship between Dynamic Forecast and Multi-Sourcing: λ_R

$\mathbb{E}K_t$	6	8	10	12	14	16	18	20	22	24
$c_1 = 20$	21.970	31.173	16.706	−5.992	−3.546	−0.614	−0.750	−0.001	0.000	0.000
$c_1 = 25$	13.573	25.906	14.590	−9.593	−9.802	−5.402	−1.061	−0.002	0.000	0.000
$c_1 = 30$	5.523	18.010	9.640	−18.377	−17.395	−8.601	−1.750	−0.002	0.000	0.000
$c_1 = 35$	6.355	16.546	−0.882	−8.894	−9.525	−5.047	−1.140	−0.002	0.000	0.000

inter-temporal substitution and inter-temporal rationing power of signal-based forecast can be extracted to a large extent via single-sourcing. In this case, the firm benefits little from adding another supplier with signal-based supply forecast. (c) When the supply-demand ratio is high, capacity is no longer constraining and the two strategies are independent, because signal-based supply forecast confers no value in this case (see section 7.2), and the only value of multi-sourcing is the cost saving effect from cheaper suppliers, which is independent of the forecasting method. (d) The capacity threshold at which the relationship switches from complementary to substitutive is decreasing in the marginal cost of the new supplier (c_1). This is because, the costlier the new supplier, the less valuable its capacity, and the lower the value of signal-based forecast with this new supplier.

7.5. Value of Discretionary Selling

We measure the value of discretionary selling by $\lambda_S = (V^{\mathrm{DF}} - \check{V}^{\mathrm{DF}})/\check{V}^{\mathrm{DF}}$, where $\check{V}^{\mathrm{DF}}$ is the optimal profit with full backlog (see section 6). We conduct two sets of experiments, focusing on three factors: demand heterogeneity (r_1/r_2), capacity coefficient of variation (CV of K_t), and capacity availability ($\mathbb{E}K_t$). We set $p_\theta = 0.75$ and $p_K = 1$.

The first experiment varies the demand heterogeneity $r_1/r_2 \in \{1.0, 1.44, 2.14\}$ while keeping $r_1 + r_2 = 110$. The results in Figures 7 and 8 show that, when the total effective marginal revenue is fixed, the value of discretionary selling λ_S is (a) increasing in demand heterogeneity, (b) increasing in capacity variability, and (c) decreasing in capacity availability. When the total effective marginal revenue is fixed, the value of discretionary selling is driven by its ability to ration customers and choose lucrative orders: The higher the demand heterogeneity, the higher the value of discretionary selling. Even when the market is homogeneous, discretionary selling still has positive value: $\lambda_S > 0$ for $(r_1, r_2) = (55, 55)$. This value comes from the capability of discretionary selling to decrease backlog costs, by rejecting excess orders that would otherwise overwhelm the system upfront.

The second test varies demand heterogeneity $r_1/r_2 \in \{1.14, 1.43, 1.71\}$ while keeping $r_2 = 35$. In this case, Figures 9 and 10 show that, surprisingly, the value of discretionary selling decreases in demand

Figure 7 Impact of Capacity Variability on λ_S $r_1 + r_2 = 110$, $(K_l, K_h) = \{(0, 12), (1, 11), \ldots, (6, 6)\}$

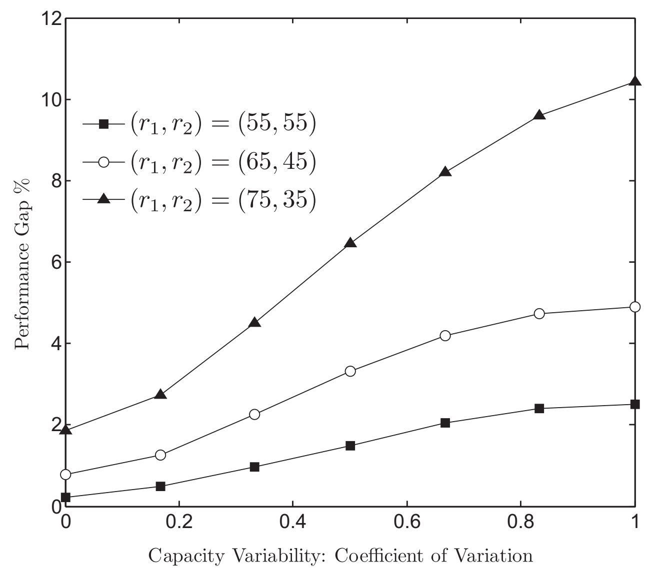
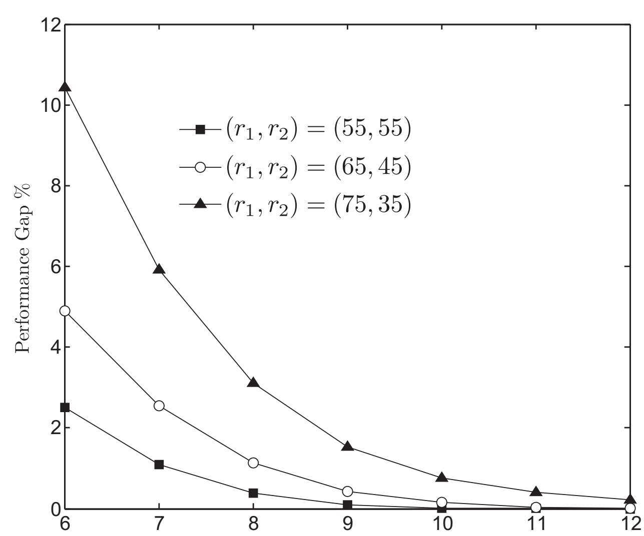


Figure 8 Impact of Capacity Mean on λ_S: $r_1 + r_2 = 110$, $(K_l, K_h) = \{(0, 12), (1, 13), \ldots, (6, 18)\}$



heterogeneity. The rationale for this counterintuitive result is that, when the effective marginal revenue from the high-value demand class increases ($r_1 \uparrow$ and, hence, the heterogeneity $r_1/r_2 \uparrow$), it also drives up bask stock levels (see Theorem 8(*iii*)), reducing the

Figure 9 Impact of Capacity Variability on λ_S: $r_2 = 35$, $(K_l, K_h) = \{(0, 12), (1, 11), \ldots, (6, 6)\}$

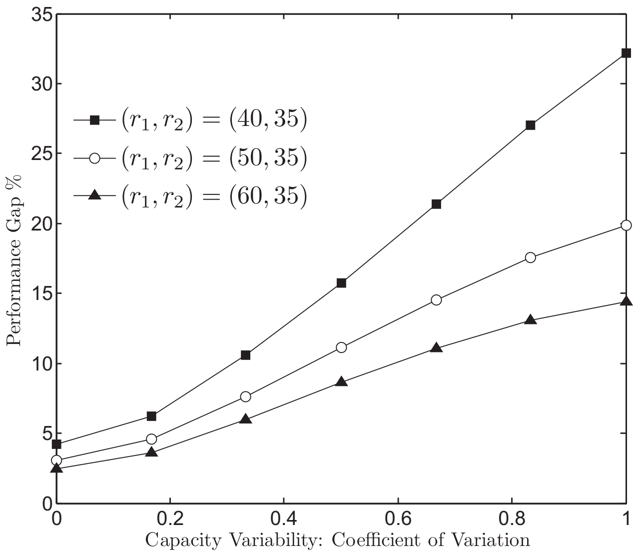
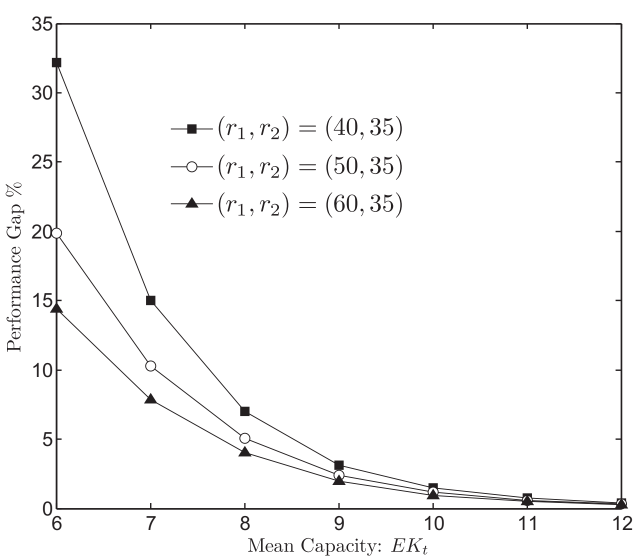


Figure 10 Impact of Capacity Mean on λ_S: $r_2 = 35$, $(K_l, K_h) = \{(0, 12), (1, 13), \ldots, (6, 18)\}$



likelihood of shortage, and, hence, the value of discretionary selling.

8. Extensions

This section extends our base model to the case with supplier and demand class dependent inventory costs in section 8.1, the case with convex rejection cost in section 8.2, and other cases in section 8.3.

8.1. Supplier and Demand Class Dependent Inventory Costs

For tractability, we assume in our base model that inventory cost $h_t(\cdot)$ is supplier and demand class independent. This independence assumption applies when inventory cost is much lower than sales revenue, or supplier and demand heterogeneities are low to moderate. When the independence assumption no longer holds, we use $h_t^i(\cdot)$ to denote the holding cost of excess inventory from supplier $i \in \mathcal{M}$ in period t, and $p_t^j(\cdot)$ for backlogging cost of demand class $j \in \mathcal{N}$. We assume that all inventory cost functions are continuously differentiable, $h_t^i(I)$ is convexly increasing for $I \geq 0$, $p_t^j(I)$ is convexly decreasing for $I \leq 0$, and $h_t^i(0) = p_t^j(0) = 0$ for all i and j.

This model needs an $(m + n)$-dimension vector to record backlogs and stocks. Let $E_t^i \geq 0$ be the starting on-hand inventory from supplier i, and $B_t^j \leq 0$ the starting backlogging level of demand class j in period t. Let $E_t := (E_t^1, E_t^2, \cdots, E_t^m)$ be the excess inventory, and $B_t := (B_t^1, B_t^2, \cdots, B_t^n)$ be the backlogged demand. Hence, the problem has state variable $(E_t, B_t, K_t, \theta_t)$ and can be formulated as

$$V_t(E_t, B_t, K_t, \theta_t) = \max\{H_t(E_t, B_t, x_t, \theta_t) : 0 \leq x_t \leq K_t\}, \tag{19}$$

$$H_t(E_t, B_t, x_t, \theta_t) = -c_t \cdot x_t + \mathbb{E}_{D_t}[W_t(E_t + x_t, B_t, D_t, \theta_t)], \tag{20}$$

$$\begin{aligned} W_t(J_t, B_t, D_t, \theta_t) = \max\{&G_t(J_t, B_t, y_t, z_t, w_t, \theta_t) \\ &- b_t \cdot D_t : 0 \leq y_t \leq D_t, 0 \leq z_t \leq J_t, \\ &w_t \geq 0, |w_t| = |z_t|\}, \end{aligned} \tag{21}$$

$$\begin{aligned} &G_t(J_t, B_t, y_t, z_t, w_t, \theta_t) \\ &= r_t \cdot y_t - \sum_{i \in \mathcal{M}} h_t^i(J_t^i - z_t^i) - \sum_{j \in \mathcal{N}} p_t^j\big(-(B_t^j - y_t^j + w_t^j)^-\big) \\ &\quad + \gamma \mathbb{E}_{(K_{t-1}, \theta_{t-1})}[V_{t-1}(J_t - z_t, -(B_t - y_t + w_t)^-, \\ &\quad K_{t-1}, \theta_{t-1})|\theta_t], \end{aligned} \tag{22}$$

where $J_t := E_t + x_t$ is the post-delivery on-hand inventory vector, $z_t := (z_t^1, z_t^2, \cdots, z_t^m) \leq J_t$ is the inventory used for current fulfillment, and $w_t := (w_t^1, w_t^2, \cdots, w_t^n) \geq 0$ is the current allocation scheme to fulfil demand. The constraint $|w_t| = |z_t|$ follows from the mass conservation law. We can show that $V_t(\cdot, \cdot, K_t, \theta_t)$, $H_t(\cdot, \cdot, \cdot, \theta_t)$, $W_t(\cdot, \cdot, D_t, \theta_t)$, and $G_t(\cdot, \cdot, \cdot, \cdot, \cdot, \theta_t)$ are jointly concave and continuously differentiable for any D_t, K_t and θ_t. However, we cannot characterize the optimal procurement and selling policy due to the curse of dimensionality, which is the major obstacle for our analysis of the model with supplier and demand class dependent inventory costs. Such obstacle also restricts our model in real applications due to the computational complexity (see, also, Gupta and Wang 2007).

In the literature, several approximation schemes are used to resolve the curse of dimensionality with class-dependent costs (Subramanian et al. 1999). The common idea (which we refer to as the expected fraction heuristic) is to estimate the individual excess inventory E_t^i from supplier i [backlog B_t^j from demand class j] based on its fraction in the total excess inventory $|E_t|$ [total backlogged demand $|B_t|$]. More specifically, let u^i [v^j] be the expected fraction of excess inventory from supplier i [backlogged demand from demand class j], where $\sum_{i\in\mathcal{M}} u^i = 1$ [$\sum_{j\in\mathcal{N}} v^j = 1$]. Assume that it incurs linear supplier-dependent holding [demand-class-dependent backlogging] cost with unit cost $\mathfrak{h}_t^i$ [$\mathfrak{p}_t^j$], we have

$$\begin{aligned}\sum_i h_t^i(E_t^i) + \sum_j p_t^j(B_t^j) &= \sum_i \mathfrak{h}_t^i E_t^i + \sum_j \mathfrak{p}_t^j(-B_t^j)\\ &= \sum_i \mathfrak{h}_t^i(u^i|E_t|) + \sum_j \mathfrak{p}_t^j(-v^j|B_t|)\\ &= (\sum_i u^i \mathfrak{h}_t^i)I_t^+ + (\sum_j v^j \mathfrak{p}_t^j)I_t^-,\end{aligned}$$

where $I_t \in \mathbb{R}$ is the total inventory level, and $\{u^i\}_{i\in\mathcal{M}}$ and $\{v^j\}_{j\in\mathcal{N}}$ are determined by historical data. This scheme provides a good approximation, as long as $\{u^i\}_{i\in\mathcal{M}}$ and $\{v^j\}_{j\in\mathcal{N}}$ can be accurately estimated from historical data.

8.2. Convex Rejection Cost

When the firm rejects more orders from the same class, the loss of goodwill may become increasingly severe. To model this phenomenon, we generalize our base model to the one with convex rejection cost. In this case, the firm pays $b_t^j(D_t^j - y_t^j)$ for rejecting $(D_t^j - y_t^j)$ from class j, where $b_t^j(\cdot)$ is a convexly increasing and continuously differentiable function. The formulation of this problem is the same as that of the base model, except for Equations (4) and (5), which are modified to:

$$W_t(J_t, D_t, \theta_t) = \max\{G_t(J_t, y_t, D_t, \theta_t) : 0 \le y_t \le D_t\}, \tag{23}$$

$$\begin{aligned}G_t(J_t,y_t,D_t,\theta_t) = \tilde{r}_t \cdot y_t - \sum_{j\in\mathcal{N}} b_t^j(D_t^j - y_t^j) - h_t(J_t - |y_t|)\\ + \gamma\mathbb{E}_{(K_{t-1},\theta_{t-1})}[V_{t-1}(J_t - |y_t|, K_{t-1}, \theta_{t-1})|\theta_t].\end{aligned} \tag{24}$$

We can show that $V_t(\cdot, K_t, \theta_t)$, $H_t(\cdot, \cdot, \theta_t)$, $W_t(\cdot, D_t, \theta_t)$, and $G_t(\cdot, \cdot, D_t, \theta_t)$ are jointly concave and continuously differentiable for any D_t, K_t and θ_t. The optimal selling policy, however, cannot be determined by the demand and inventory independent demand rationing levels, because the convex rejection cost makes the demand inseparable from the objective function (24). All other results and insights of this study, such as the structure of the optimal procurement policy, the impact of discretionary selling upon the base stock levels, the value of signal-based dynamic supply forecast, and the strategic relationship between signal-based forecast and supply diversification, remain valid in the model with convex rejection cost.

8.3. Other Extensions

Our model extends to the Markov modulated demand case. Specifically, let $\eta_t = (\eta_t^1, \eta_t^2, \cdots, \eta_t^n)$ be the *advance demand signal* in period t. The class j demand $D_t^j(\eta_t^j)$ now stochastically depends on η_t^j, where $D_t^j(\eta_t^j)$ is stochastically increasing in η_t^j. For this model with signal-based forecast in both demand and supply, all our results in this study remain valid.

Our base model assumes that, in each period, the procurement decision is made after capacity realization. This assumption applies when the production leadtime is short. It is consistent with part of the random capacity literature (Chao et al. 2008) and the random supply disruption literature (Aydin et al. 2011), in which the firm knows the supply state before making decisions in each period. If this assumption is relaxed, the resulting non-concave objective functions (see, e.g., Ciarallo et al. 1994) make the problem prohibitively difficult to analyze; in particular, the nested threshold structure of the optimal procurement policy breaks down. This is a challenging avenue for future research.

Random yield risk arises when the unreliable suppliers can fulfil only a fraction of the orders. Specifically, given procurement decision x_t and signal θ_t, the actual delivery by supplier i is $x_t^i\epsilon_t^i(\theta_t^i)$, where $\epsilon_t^i(\theta_t^i)$ is the random yield factor of supplier i with support on [0, 1]. If $\epsilon_t(\theta_t) := (\epsilon_t^1(\theta_t^1), \epsilon_t^2(\theta_t^2), \cdots, \epsilon_t^m(\theta_t^m))$ realizes before the procurement decision in period t, the problem reduces to the base model with modified procurement costs, and all the results of our base model continue to hold. If $\epsilon_t(\theta_t)$ realizes after the procurement decision in period t, the structure of the optimal selling policy still holds, but the optimal procurement policy no longer has the nested threshold structure.

Our base model assumes that customers pay the sales prices upon order acceptance. This assumption is common in inventory management models (e.g., Federgruen and Heching 1999, Zipkin 2000). In this case, the underage penalty is summarized by the backlogging cost. Now consider the case where customers pay upon order delivery. If customers pay the discount-adjusted price, that is, class j customers pay unit price $\tilde{r}_t^j/\gamma^{t-s}$ for the orders accepted in period t and delivered in period s ($s \le t$), the problem is equivalent to the base model. If those customers pay the price $\tilde{r}_t^j$ in period s, the problem becomes more involved, because we have to record backlogged demand by both demand class and order time. In this case, the optimal policy no longer has the nested threshold structure.

9. Concluding Remarks

In this paper, we study a dynamic supply risk management problem with signal-based supply forecast, multi-sourcing, and discretionary selling. We develop a hierarchical Markov model to capture the stochastic attributes of the ever-changing real-time supply information, and integrate it with procurement and selling decisions for dynamic risk management. Despite the problem complexity, we pin down the optimal policy by two sequences of signal-dependent monotone thresholds. The optimal procurement is driven by multi-sourcing and inter-temporal substitution; the optimal selling is driven by customer segmentation and inter-temporal rationing; and they are synchronized by dynamic forecast for adaptive and resilient risk mitigation. We demonstrate that the traditional stationary forecast for uncertain supply capacity, which ignores the volatility of supply capacity evolution, may generate misleading recommendations, especially for the case with moderate supply-demand ratio. As this situation is prevalent in business practice, our result calls for caution on ignoring supply volatility: It can inflict severe losses. Our analysis also helps rationalize several counterintuitive insights. Numerically, we show that demand heterogeneity may *reduce* the value of discretionary selling, that multi-sourcing and dynamic forecast can be either strategic complements or substitutes, and that the mean forecast may outperform the stationary forecast.

We demonstrate the critical role of signal-based dynamic forecast in supply risk management. We show how signal-based dynamic supply forecast influences the procurement and selling policies: Advance supply signals contain rich information on *future* supply risks and, in turn, guide the *current* base stock and demand rationing levels. Moreover, we demonstrate that signal-based forecast is most valuable when: (a) Capacity uncertainty is high so that the supply risk is substantial, (b) supply-demand ratio is moderate so that meticulous management matters a great deal, (c) forecast precision is high so that advance supply signals confer great value, and (d) the firm has diverse suppliers so that the operational levers are effective.

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Supporting Information

Additional supporting information may be found online in the supporting information tab for this article:

Appendix S1: Table of Notations.
Appendix S2: Concavity and Supermodularity.
Appendix S3: Proofs.

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