Dynamic Pricing and Inventory Management: Theory and Applications

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Dissertation Defense

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Motivation

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  - Price;
  - Inventory.
- Dynamic, uncertain, and (possibly) competitive market environment.
- Emerging trends in
  - Technology and marketplace (e.g., social networks, customer behaviors);
  - Society (e.g., sustainability concerns).

Running Questions of Interest

- How would these emerging market trends influence the operations decisions and profitability of a firm?

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- What pricing and inventory strategies could a firm use to leverage these trends?

Outline

- Network externalities (monopoly setting) (Yang and Z, 2015a)
- Network externalities (oligopoly setting) (Yang and Z, 2015b)
- Trade-in remanufacturing (Zhang and Z, 2015)
- Scarcity effect of inventory (Yang and Z, 2014)
- Comparative statics method (Yang and Z, 2016)
- Conclusion

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Network Externalities

- Dynamic Pricing and Inventory Management under Network Externalities. (Yang and Z, 2015a)

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- Customers are more willing to purchase an Xbox if there are more players on Xbox Live.
- Microsoft's strategies:
  - 50$ discount for Xbox buyers who guarantee to join Xbox live for 1 year (Tech. Times 2015).
  - 33% discount for Xbox live gold membership in Feb. 2015 (Geek Wire 2015).

Network Externalities

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- Social networks make network externalities everywhere.

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- What is the impact of network externalities upon a firm's price and inventory policy?

- What strategies can a firm use to leverage network externalities?

Literature Review

- Network economics:
  - Compatibility and technology adoption (Katz and Shapiro 1985, 1986); financial market (Diamond 1982); pricing (Dhebar and Oren 1986); network structure (Ballester et al. 2006, Chen and Zhou 2013,2015).
- Joint pricing and inventory management:
  - Single-period (Petruzzi and Dada 1999); multi-period (Federgruen and Heching 1999); fixed ordering cost (Chen and Simchi-Levi 2004a, 2004b, 2006); random yield risk (Li and Zheng 2006); lost-sales (Huh and Janakrman 2008).
- Inventory management with intertemporal demand correlations:
  - Myopic policy (Johnson and Thompson 1975); non-stationary demand (Graves 1999); joint forecasting and replenishment (Aviv 2002).

Model Setup

- $T-$period stochastic inventory system, labeled backwards $\{T, T-1, \cdots, 1\}$, full backlog.
- Objective: maximize the total expected discounted profit.
- Dynamic price and inventory adjustments.
- Purchasing cost c, holding cost h, backlogging cost b, and discount factor α.

Model: Network Externalities

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  - N_t: network size in period t.
  - $r_n(\cdot)$: per-period profit from the network, concavely increasing in the network size.

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- $V + \gamma(N_t)$: willingness-to-pay of a customer (Katz and Shapiro 1985).
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Demand in period t

$$D_t(p_t, N_t) = \bar{V}_t + \gamma(N_t) - p_t + \xi_t.$$

- $\{\xi_t\}$: $i.i.d.$ continuously distributed demand perturbations with $\mathbb{E}[\xi_t] = 0$.
- $D_t(p_t, N_t) \geq 0$ for all N_t and p_t.

Model: Network Size Dynamics

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- Inventory carried over to the next period; network size updated.

Dynamic Program Formulation

$v_t(I_t, N_t) =$the maximal expected discounted profit in periods $t, t-1, \cdots, 1,$
with starting inventory level I_t and network size N_t in period t.

Terminal condition: $v_0(I_0, N_0) = cI_0$.

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Bellman Equation

$$v_t(I_t, N_t) = cI_t + \max_{x_t \geq I_t, p_t \in [\underline{p}, \bar{p}]} J_t(x_t, p_t, N_t), \text{ where}$$

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$$\underbrace{+\alpha v_{t-1}(x_t - D_t(p_t, N_t), N_{t-1})}_{\text{Future profit}} | N_t\}.$$

Optimal Policy

- $(x_t^*(I_t, N_t), p_t^*(I_t, N_t))$: the optimal decisions in period t.
- The network-size-dependent base-stock/list-price policy is optimal:
  - If $I_t \leq x_t(N_t)$, order up to $x_t(N_t)$ and charge a list price $p_t(N_t)$.
  - If $I_t > x_t(N_t)$, order nothing and charge an inventory-dependent price.
  - $x_t(N_t) > 0$.

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We can reduce the state space dimension from two to one.

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Theorem

- If $I_T \leq x_T(N_T)$, $(x_t^*(I_t, N_t), p_t^*(I_t, N_t)) = (x_t(N_t), p_t(N_t))$ with probability 1.
- The optimal base-stock level and list price $(x_t(N_t), p_t(N_t))$ is the solution to the following dynamic program with a 1-dimensional state space:

$$\pi_t(N_t) = \max_{x_t \geq 0, p_t \in [\underline{p}, \bar{p}]} J_t(x_t, p_t, N_t), \text{ where}$$

$$J_t(x_t, p_t, N_t) = R_t(p_t, N_t) + \beta x_t + \Lambda(x_t - \bar{V}_t + p_t - \gamma(N_t)) + G_t(\theta(\bar{V}_t - p_t + \gamma(N_t)) + \eta N_t),$$

$$\text{with } G_t(y) := \mathbb{E}\{r_n(y + \theta\xi_t + \epsilon_t) + \alpha\pi_{t-1}(y + \theta\xi_t + \epsilon_t)\}, \text{ and } \pi_0(\cdot) \equiv 0.$$

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Intuitions:

- For all N_t and N_{t-1}, $\mathbb{P}[x_t(N_t) - D_t(p_t(N_t), N_t) \le x_{t-1}(N_{t-1})] = 1$.
- As long as $I_T \le x_T(N_T)$, $I_t \le x_t(N_t)$ for all t with probability 1.

Managerial Implications of Network Externalities

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Compared with the benchmark case without NE,

(a) $x_t(N_t)$ is higher with the presence NE.

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(b) There exists a threshold $\mathfrak{N}_t$, such that

  (i) $p_t(N_t)$ is lower with the presence of NE if $N_t < \mathfrak{N}_t$.

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- Network externalities lead to higher demand/base-stock level.
- Impact of network externalities on the pricing policy:
  - A lower price to induce future demands with a small network size.
  - A higher price to exploit the better market condition with a large network size.

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If the market stationary ($\bar{V}_T = \bar{V}_{T-1} = \cdots = \bar{V}_2 = \bar{V}_1$),

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- high network externalities intensity;
- high proportion of social customers;
- high network size carry-through rate.

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- In period t, adopts the pricing and inventory policy that maximizes the profit in periods $\{t, t-1, t-2, t-3, t-4\}$.
- Achieves an optimality loss of less than 2%.

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- Achieves an optimality loss of less than 2%.
- It suffices to balance generating profits and inducing demands in the near future.

Takeaways

- State space dimension reduction.
- Tradeoff: generating current profits and inducing future demands.
- Effective strategies to exploit network externalities.

Network Externalities: Oligopoly Setting

- Dynamic Competition under Market Size Dynamics: Balancing the Exploitation-Induction Tradeoff. (Yang and Z, 2015b)

Dynamic Competition under Network Externalities

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- Generating current profits v.s. winning future market shares.
  - Exploitation-induction tradeoff.

Main Findings

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  - Inventory dynamics do not affect the equilibrium outcome.

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- Exploitation-induction tradeoff:
  - Captured by a linear coefficient of market size.
  - When the coefficient is larger, price decreases and base-stock level increases.

Trade-in Remanufacturing

- Trade-in Remanufacturing, Strategic Customer Behavior, and Government Subsidies. (Zhang and Z, 2015)

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- "We recovered enough steel in 2014 that the equivalent could be used to build over 100 miles of railroad track."

Strategic Customer Behavior

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- Value of trade-in recycling/remanufacturing under different customer behaviors:
  - To the firm;
  - To the environment.
  - To the society.

Questions of Interest

- What is the value of trade-in remanufacturing to the firm and the environment under different customer behaviors?

- How should the government design the public policy that can induce the socially optimal outcome?

Literature

- Sustainable operations and remanufacturing:
  - Inventory control (Van der Laan et al. 1999); reverse channel structure (Savaskan et al. 2004); trade-in program (Ray et al. 2005); environmental impact (Agrawal et al. 2012).
- Strategic customer behavior:
  - Pricing (Bensako and Winston 1990); availability (Su and Zhang 2008); capacity rationing (Liu and Van Ryzin 2008); quick response (Cachon and Swinney 2009); product launches (Lobel et al. 2015).
- Trade-in rebates with forward-looking customers:
  - Price commitment (Van Ackere and Reyniers 1995); two product generations (Fudenberg and Tirole 1998); lemon problem (Rao et al. 2009).

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Parameters on Environmental Impact:

- $\kappa_1 =$ unit first-generation (negative) life-cycle environmental impact.
- $\iota_1 =$ unit environmental benefit of remanufacturing ($\iota_1 < \kappa_1$).

Model: Second Period

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- The firm remanufactures the used first-generation products.

Solution Approach

- Customer behaviors: Strategic customers v.s. Myopic customers.
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Theorem

An RE equilibrium exists.

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- Strategic customers anticipate the potential price discount.
- Remanufacturing ensures such discount is high enough.
- The firm may charge a higher price with strategic customers.

Value of Trade-in Remanufacturing to the Firm

- Revenue generating effect.
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Numerical Results

	Min	5th pctile	Median	95th pctile	Max	Mean
Strategic Customers	5.8	11.3	28.3	55.8	61.6	30.2
Myopic Customers	0.008	0.22	2.5	8.1	11.7	3.1

Table: Profit Improvements of Trade-in Remanufacturing (%)

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Strategic Customers	-1.2	2.0	37.8	117.8	171.9	49.2
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Table: Environmental Impact Increases of Trade-in Remanufacturing (%)

Interactions between Strategic Customer Behavior and Trade-in Remanufacturing

- Good news about strategic customer behavior.

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- Most natural subsidization policy:
  - Subsidizes for remanufactured products only.
  - Leads to undesired outcomes.

Socially Optimal Government Policy

- Government subsidy/tax scheme $s_g = (s_1, s_2, s_r)$.
  - s_1 = per unit subsidy/tax for first-generation products.
  - s_2 = per unit subsidy/tax for second-generation products.
  - s_r = per unit subsidy/tax for *remanufacturing* in both periods.

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With strategic (myopic) customers, a linear subsidy/tax scheme $s_g^* = (s_1^*, s_2^*, s_r^*)$ ($\tilde{s}_g^* = (\tilde{s}_1^*, \tilde{s}_2^*, \tilde{s}_r^*)$) can induce the social optimum.

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- Implications:
  - The government should subsidize/tax both product generations and remanufacturing.
  - A linear subsidy/tax scheme can induce the social optimum.

Takeaways

- Value of trade-in remanufacturing to the firm and the environment:
  - Benefit of strategic customer behavior to the firm.
  - Tension between firm profitability and environmental sustainability.

- Socially optimal government policy:
  - Subsidies/taxes for both new and remanufactured products.
  - A simple linear subsidy/tax scheme to induce the social optimum.

Scarcity Effect of Inventory

- Dynamic Pricing and Inventory Management under Inventory-Dependent Demand. (Yang and Z, 2014, *Operations Research*)

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- Inventory level reveals the pricing strategy of the firm.
- "Scarcity strategy": a basic tactic for modern marketers. (Dye 2000, Brown 2001).

Main Findings

- Optimal policy: a customer-accessible-inventory dependent order-up-to/dispay-up-to/dispose-down/list-price policy.

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- The scarcity effect of inventory strengthens overstocking risk.

- Price and operational flexibilities help mitigate demand loss driven by high inventory levels.

Comparative Statics Analysis Method

- Comparative Statics Analysis Method of Inventory Management Models with Dynamic Pricing. (Yang and Z, 2016)

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- Comparisons between decisions $\longrightarrow$ comparisons between partial derivatives.
- Features of the new method:
  - Non-restrictive conditions;
  - Scalable;
  - Some optimal decisions can be non-monotone.

Conclusion

- How to optimize the price and inventory decisions?
  - Network externalities: Monopoly setting.
  - Network externalities: Oligopoly setting.
  - Trade-in remanufacturing.
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- Important and interesting implications of the emerging trends in technology, marketplace and society.

Q&A

Thank you!

Questions?