Appendix A: Table of Notations

Table 4 Summary of Notations

X :	market size (total number of potential customers)	c_2 :	unit production cost of 1st-generation product
$F(\cdot)$:	distribution function of X	κ_2 :	unit environmental impact of 2nd-generation product
X_1 :	realized demand in period 1	r_2:	unit net revenue of remanufacturing for firm
X_2^n :	market size of new customers in period 2	ι_2:	unit environmental benefit of remanufacturing
X_2^r :	market size of repeat customers in period 2	e_2:	unit total benefit of remanufacturing, $e_2 = r_2 + \iota_2$
V :	customer valuation for 1st-generation product	p_1:	price for 1st-generation product
$G(\cdot)$:	distribution function of V, $\bar{G}(\cdot) = 1 - G(\cdot)$	Q_1:	production quantity in period 1
$g(\cdot)$:	density function of V	p_2^n :	price for new customers in period 2
$h(\cdot)$:	hazard rate function of V, i.e., $h(v) = g(v)/\bar{G}(v)$	p_2^r :	price for repeat customers in period 2
α:	innovation level of 2nd-generation product	Q_2^n :	production quantity for new customers in period 2
k :	product depreciation	Q_2^r :	production quantity for repeat customers in period 2
c_1:	unit production cost of 1st-generation product	δ :	discount factor for firm
κ_1:	unit environmental impact of 1st-generation product	δ_c :	discount factor for customers

Appendix B: Auxiliary Results

In this section, we present some auxiliary results in the NTR model and the model of social optimum. These results are building blocks of our subsequent analysis. The proofs of these results are available from the authors upon request. To begin with, we characterize the second-period equilibrium pricing and production strategy in the NTR model. Let $Q_u^n(X_2^n, X_2^r)$ and $Q_u^r(X_2^n, X_2^r)$ be the equilibrium production quantities for new and repeat customers, respectively.

LEMMA 2. *(a) For any (X_2^n, X_2^r), $p_2^u(X_2^n, X_2^r) = \arg\max_{p_2^u \geq 0} \Pi_2^u(p_2^u|X_2^n, X_2^r)$, where $\Pi_2^u(p_2^u|X_2^n, X_2^r) := X_2^n(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{1+\alpha}\right) + X_2^r(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{k+\alpha}\right)$.*

(b) For any (X_2^n, X_2^r), $Q_u^n(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) X_2^n$, and $Q_u^r(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right) X_2^r$.

(c) $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r. Moreover, for any (X_2^n, X_2^r), $p_2^{r*} \leq p_2^u(X_2^n, X_2^r) \leq p_2^{n*}$, where the inequalities are strict if $X_2^n, X_2^r > 0$..

Let $\Pi_f^u(Q_1|\delta_c)$ ($p_1^u(Q_1|\delta_c)$) be the expected profit (equilibrium first-period price) of the firm to produce Q_1 products in period 1 in the NTR model with customer discount factor δ_c. We compute $\Pi_f^u(\cdot|\cdot)$ in the following lemma.

LEMMA 3. *In the NTR model, we have $p_1^u(Q_1|\delta_c) = \mu + \delta_c(\sigma_r^u(Q_1) - \sigma_n^u(Q_1))$ and $\Pi_f^u(Q_1|\delta_c) = (m_1^u(Q_1|\delta_c) - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s)Q_1 + \delta R_2^u(Q_1)$, where $m_2^u(Q_1|\delta_c) = \mu + \delta(\beta_r^u(Q_1) - \beta_n^u(Q_1)) + \delta_c(\sigma_r^u(Q_1) - \sigma_n^u(Q_1))$, $\beta_n^u(Q_1) := \mathbb{E}[\hat{v}_2^r(p_2^u(X_2^n, X_2^r))]$, $\beta_n^u(Q_1) := \mathbb{E}[\hat{v}_2^r(p_2^u(X_2^n, X_2^r))]$, and $R_2^u(Q_1) = \mathbb{E}[v_2^n(p_2^u(X_2^n, X_2^r))X]$ ($X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$). Moreover, $\beta_r^u(\cdot)$ is increasing, whereas $\sigma_r^u(\cdot)$, $\sigma_n^u(\cdot)$, $\beta_n^u(\cdot)$ and $R_2^u(\cdot)$ are decreasing in Q_1, respectively.*

It is clear that $\beta_n^u(Q_1)$ and $\beta_r^u(Q_1)$ are the expected second-period unit profit from new and repeat customers in the NTR model, respectively, whereas $m_2^u(Q_1|\delta_c)$ is the effective first-period marginal revenue. $\beta_n^u(\cdot)$, $\beta_r^u(\cdot)$, and $m_1^u(\cdot|\cdot)$ are the counterparts of β_n^*, β_r^*, and $m_1(\cdot)$ in the NTR model. The following theorem summarizes the equilibrium price and production quantity $(p_1^{u*}(\delta_c), Q_1^{u*}(\delta_c))$ in the NTR model.

Theorem 11. *In the NTR model, for any customer discount factor δ_c, a unique RE equilibrium exists with (a) $Q_1^{u*}(\delta_c) = \arg\max_{Q_1 \geq 0} \Pi_f^u(Q_1|\delta_c)$; (b) $p_1^{u*}(\delta_c) = \mu + \delta_c(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)))$; and (c) the expected profit of the firm $\Pi_f^{u*}(\delta_c) = (m_1^u(Q_1^{u*}(\delta_c)|\delta_c) - s)\mathbb{E}(X \wedge Q_1^{u*}(\delta_c)) - (c_1 - s)Q_1^{u*}(\delta_c) + \delta R_2^u(Q_1^{u*}(\delta_c))$.*

Finally, we have the following lemma that characterizes the equilibrium second-period pricing strategy in the model of social optimum.

Lemma 4. *(a) $p_s^n(X_2^n, X_2^r) \equiv p_s^{n*}$ and $p_s^r(X_2^n, X_2^r) \equiv p_s^{r*}$, where $p_s^{n*} = c_2 + \kappa_2$ and $p_s^{r*} = c_2 - r_2 + \kappa_2 - \iota_2$. Hence, $p_s^{n*} > p_s^{r*}$ if and only if $r_2 > 0$ or $\iota_2 > 0$.*

(b) $w_2(X_2^n, X_2^r) = \sigma_n^{s*}X_2^n + \sigma_r^{s*}X_2^r$, where $\sigma_n^{s*} = \mathbb{E}((1+\alpha)V - p_s^{n*})^+$ and $\sigma_r^{s*} = \mathbb{E}((k+\alpha)V_2 - p_s^{r*})^+$.

Appendix C: Proofs of Statements

[1]**Proof of Lemma 1: Part (a).** Given (p_2^n, p_2^r) with $p_2^r \leq p_2^n$, the *ex-ante* probability that a new customer will purchase the second-generation product is $\bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$, whereas the probability that a repeat customer will join the trade-in program is $\bar{G}\left(\frac{p_2^r}{k+\alpha}\right)$. Therefore, conditioned on the realized market size (X_2^n, X_2^r), the expected profit of the firm in period 2 is given by: $\Pi_2(p_2^n, p_2^r|X_2^n, X_2^r) := X_2^n(p_2^n - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right) + X_2^r(p_2^r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right) = X_2^n v_2^n(p_2^n) + X_2^r v_2^r(p_2^r)$, where $v_2^n(p_2^n) := (p_2^n - c_2)\bar{G}(\frac{p_2^n}{1+\alpha})$ and $v_2^r(p_2^r) := (p_2^r - c_2 + r_2)\bar{G}(\frac{p_2^r}{k+\alpha})$. We now show that $v_2^n(\cdot)$ is quasiconcave in p_2^n, and $v_2^r(\cdot)$ is quasiconcave in p_2^r. Note that $\partial_{p_2^n} v_2^n(p_2^n) = -\left(\frac{p_2^n - c_2}{1+\alpha}\right) g\left(\frac{p_2^n}{1+\alpha}\right) + \bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$ and $\partial_{p_2^r} v_2^r(p_2^r) = -\left(\frac{p_2^r - c_2 + r_2}{k+\alpha}\right) g\left(\frac{p_2^r}{k+\alpha}\right) + \bar{G}\left(\frac{p_2^r}{k+\alpha}\right)$. Because $h(v) = g(v)/\bar{G}(v)$ is continuously increasing in v, $g(\frac{p_2^n}{1+\alpha})/\bar{G}(\frac{p_2^n}{1+\alpha})$ is continuously increasing in p_2^n and $g(\frac{p_2^r}{k+\alpha})/\bar{G}(\frac{p_2^r}{k+\alpha})$ is continuously increasing in p_2^r. Hence, $\partial_{p_2^n} v_2^n(p_2^n) = 0$ has a unique solution p_2^{n*} and $\partial_{p_2^r} v_2^r(p_2^r) = 0$ has a unique solution p_2^{r*}. Clearly, for $i = n, r$, $v_2^i(\cdot)$ is strictly increasing on $[0, p_2^{i*})$ and strictly decreasing on $(p_2^{i*}, +\infty)$. Therefore, $\Pi_2(\cdot, \cdot|X_2^n, X_2^r)$ is quasiconcave in (p_2^n, p_2^r), and $(p_2^n(X_2^n, X_2^r), p_2^r(X_2^n, X_2^r)) = (p_2^{n*}, p_2^{r*})$.

It remains to show that $p_2^{n*} > p_2^{r*}$. Note that p_2^{n*} satisfies $\left(\frac{p_2^{n*} - c_2}{1+\alpha}\right) g\left(\frac{p_2^{n*}}{1+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) = 1$, and p_2^{r*} satisfies $\left(\frac{p_2^{r*} - c_2 + r_2}{k+\alpha}\right) g\left(\frac{p_2^{r*}}{k+\alpha}\right) \Big/ \bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) = 1$. Since $k < 1$, $\frac{p_2^{n*} - c_2 + r_2}{k+\alpha} > \frac{p_2^{n*} - c_2}{1+\alpha}$, and the increasing failure rate condition implies that $g\left(\frac{p_2^{n*}}{k+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right) \geq g\left(\frac{p_2^{n*}}{1+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$. Thus, $\left(\frac{p_2^{n*} - c_2 + r_2}{k+\alpha}\right) g\left(\frac{p_2^{n*}}{k+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right) > \left(\frac{p_2^{n*} - c_2}{1+\alpha}\right) g\left(\frac{p_2^{n*}}{1+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) = 1$, and, hence, $\partial_{p_2^r} v_2^r(p_2^{n*}) < 0$. Since $v_2^r(\cdot)$ is quasiconcave, $p_2^{r*} < p_2^{n*}$.

Part (b). Because all new customers with willingness-to-pay $(1+\alpha)V$ greater than $p_2^n(X_2^n, X_2^r) \equiv p_2^{n*}$ would make a purchase. Hence, $Q_2^n(X_2^n, X_2^r) = \mathbb{E}[X_2^n 1_{\{(1+\alpha)V \geq p_2^{n*}\}}|X_2^n] = \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) X_2^n$. Analogously, all repeat customers with willingness-to-pay $(k+\alpha)V$ greater than $p_2^r(X_2^n, X_2^r) \equiv p_2^{r*}$ would make a purchase. Hence, $Q_2^r(X_2^n, X_2^r) = \mathbb{E}[X_2^r 1_{\{(k+\alpha)V \geq p_2^{r*}\}}|X_2^r] = \bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) X_2^r$.

Part (c). Since $\pi_2(X_2^n, X_2^r) := \max\{\Pi_2(p_2^n, p_2^r|X_2^n, X_2^r) : 0 \leq p_2^r \leq p_2^n\}$, it follows that $\pi_2(X_2^n, X_2^r) = [\max v_2^n(p_2^n)]X_2^n + [\max v_2^r(p_2^r)]X_2^r$. To complete the proof, it remains to show that $\beta_n^* = [\max v_2^n(p_2^n)] > 0$ and $\beta_r^* = [\max v_2^r(p_2^r)] > 0$. It is straightforward to check that $p_2^{n*} - c_2 > 0$, $\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) > 0$, $p_2^{r*} - c_2 + r_2 > 0$, and $\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) > 0$. Hence, $\beta_n^* = (p_2^{n*} - c_2)\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) > 0$ and $\beta_r^* = (p_2^{r*} - c_2 + r_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) > 0$. *Q.E.D.*

Proof of Theorem 1: Part (a). This part has already been shown by the discussions before the theorem.

[1] Due to the page limit requirement, we only provide a sketch of the proof. The complete proof is available from the authors upon request.

Part (b,c). Plugging $p_1^*(\cdot)$ into $\Pi_f(\cdot|\cdot)$ and, with some algebraic manipulations, we have $\Pi_f(Q_1|\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s)Q_1 + \delta\beta_n^*\mathbb{E}(X)$. Therefore, $Q_1^*(\delta_c)$ is the solution to a newsvendor problem with marginal revenue $m_1^*(\delta_c) - s$, marginal cost $c_1 - s$, and demand distribution $F(\cdot)$. Hence, $Q_1^*(\delta_c) = \bar{F}^{-1}(\frac{c_1-s}{m_1^*(\delta_c)-s})$ and $\Pi_f^*(\delta_c) = \Pi_f(Q_1^*(\delta_c)|\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}(X \wedge Q_1^*(\delta_c)) - (c_1 - s)Q_1^*(\delta_c) + \delta\beta_n^*\mathbb{E}(X)$. *Q.E.D.*

Proof of Theorem 2: Part (a). It follows from Theorem 1(a) that $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$ and $m_1^*(\delta_c) = \mu + \delta(\beta_r^* - \beta_n^*) + \delta_c(\sigma_r^* - \sigma_n^*)$ are strictly increasing (decreasing) in δ_c if $\sigma_r^* > \sigma_n^*$ ($\sigma_r^* < \sigma_n^*$). By Theorem 1(b), $Q_1^*(\delta_c) = \bar{F}^{-1}(\frac{c_1-s}{m_1^*(\delta_c)-s})$ is increasing (decreasing) in δ_c if and only if $\sigma_r^* > \sigma_n^*$ ($\sigma_r^* < \sigma_n^*$). Moreover, for any Q_1 and any $\hat{\delta}_c > \delta_c$, $\Pi_f(Q_1|\hat{\delta}_c) - \Pi_f(Q_1|\delta_c) = (\hat{\delta}_c - \delta_c)(\sigma_r^* - \sigma_n^*)\mathbb{E}(X \wedge Q_1) > 0$ if and only if $\sigma_r^* > \sigma_n^*$. Therefore, $\Pi_f^*(\hat{\delta}_c) = \max \Pi_f(Q_1|\hat{\delta}_c) > \max \Pi_f(Q_1|\delta_c) = \Pi_f^*(\delta_c)$ if and only if $\sigma_r^* > \sigma_n^*$. If, on the other hand, $\sigma_r^* < \sigma_n^*$, it follows immediately from the same argument that $\Pi_f^*(\hat{\delta}_c) < \Pi_f^*(\delta_c)$.

To show that $\sigma_r^* > \sigma_n^*$ (resp. $\sigma_r^* < \sigma_n^*$) if $k \in (\underline{k}, \bar{k})$ (resp. $k < \underline{k}$ or $k > \bar{k}$), it suffices to prove that if σ_r^* is increasing in k at $k = k_0$, it is increasing in k when $k \leq k_0$. σ_r^* is increasing in k at $k = k_0$ implies that, for $\epsilon > 0$ and small enough, $\mathbb{E}[(k_0 + \alpha)V - p_2^{r*}(k_0)]^+ > \mathbb{E}[(k_0 - \epsilon + \alpha)V - p_2^{r*}(k_0 - \epsilon)]^+$, where we use $p_2^{r*}(\cdot)$ to denote the dependence of p_2^{r*} on the depreciation factor k. Since r_2 is concavely decreasing in k, $p_2^{r*}(k) - p_2^{r*}(k - \epsilon)$ is increasing in k. Therefore, for $k < k_0$, $\mathbb{E}[(k + \alpha)V - p_2^{r*}(k)]^+ > \mathbb{E}[(k - \epsilon + \alpha)V - p_2^{r*}(k - \epsilon)]^+$ for $\epsilon > 0$ small enough, where the inequality follows from $\mathbb{E}[(k_0 + \alpha)V - p_2^{r*}(k_0)]^+ > \mathbb{E}[(k_0 - \epsilon + \alpha)V - p_2^{r*}(k_0 - \epsilon)]^+$ and that $p_2^{r*}(k) - p_2^{r*}(k - \epsilon)$ is increasing in k. Therefore, σ_r^* is increasing in k for all $k \leq k_0$. As an implication, we have also established that σ_r^* is quasiconcave in k. Hence, there exist two thresholds $\underline{k}$ and $\bar{k}$, such that $\sigma_r^* > \sigma_n^*$ if and only if $k \in (\underline{k}, \bar{k})$, and $\sigma_r^* < \sigma_n^*$ if and only $k < \underline{k}$ or $k > \bar{k}$.

Part (b). We first show (b-iii). By definition, $\sigma_r^u(Q_1) - \sigma_n^u(Q_1) = \mathbb{E}[(k + \alpha)V - p_2^u(X_2^n, X_2^r)]^+ - \mathbb{E}[(1 + \alpha)V - p_2^u(X_2^n, X_2^r)]^+$. Since $k < 1$ and $p_2^u(X_2^n, X_2^r) \in (p_2^{r*}, p_2^{n*})$ (Lemma 2), $\sigma_r^u(Q_1) - \sigma_n^u(Q_1) < 0$ for all $Q_1 \geq 0$.

By Theorem 11, $p_1^{u*}(\delta_c) = \mu + \delta_c(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)))$ is continuously differentiable in δ_c. Since the right derivative of $p_1^{u*}(\cdot)$ at 0 is $\partial_{\delta_c}^+ p_1^{u*}(0) = \sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)) < 0$, there exists a positive threshold $\delta_0 > 0$ such that $p_1^{u*}(\cdot)$ is strictly decreasing on $[0, \delta_0]$.

To show that $Q_1^{u*}(\delta_c)$ is strictly decreasing in δ_c, it suffices to show that $\Pi_f^u(Q_1|\delta_c)$ is strictly submodular on a neighborhood of $(Q_1^{u*}(\delta_c), \delta_c)$. Direct computation yields $\partial_{\delta_c}\Pi_f^u(Q_1(\delta_c)|\delta_c) = (\sigma_r^u(Q_1) - \sigma_n^u(Q_1))\mathbb{E}(X \wedge Q_1)$. Note that $\sigma_r^u(Q_1) - \sigma_n^u(Q_1) < 0$ and is decreasing in Q_1, whereas $\mathbb{E}(X \wedge Q_1) > 0$ and is strictly increasing in Q_1 in a neighborhood of $Q_1^{u*}(\delta_c)$. It follows immediately that $\partial_{\delta_c}\Pi_f^u(Q_1|\delta_c)$ is strictly decreasing in Q_1 on a neighborhood of $Q_1^{u*}(\delta_c)$. Therefore, $\Pi_f^u(Q_1|\delta_c)$ is strictly submodular on a neighborhood of $(Q_1^{u*}(\delta_c), \delta_c)$ and, thus, $Q_1^{u*}(\delta_c)$ is strictly decreasing in δ_c. By the envelope theorem, $\partial_{\delta_c}\Pi_f^{u*}(\delta_c) = (\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)))\mathbb{E}(X \wedge Q_1^{u*}(\delta_c)) > 0$. Hence, $\Pi_f^{u*}(\delta_c)$ is strictly increasing in δ_c. *Q.E.D.*

Proof of Theorem 3: Part (a). By Lemma 2, $p_2^{r*} < p_2^u(X_2^n, X_2^r) < p_2^{n*}$ with probability 1. Thus, if $Q_1 > 0$, $\sigma_r^* = \mathbb{E}[(k + \alpha)V - p_2^{r*}]^+ \geq \mathbb{E}[(k + \alpha)V - p_2^u((X - Q_1)^+, X \wedge Q_1)]^+ = \sigma_r^u(Q_1)$, and $\sigma_n^* = \mathbb{E}[(1 + \alpha)V - p_2^{n*}]^+ < \mathbb{E}[(1 + \alpha)V - p_2^u((X - Q_1)^+, X \wedge Q_1)]^+ = \sigma_n^u(Q_1)$.

Part (b). By Theorem 1(b) and Theorem 11(b), for all $\delta_c > 0$, $p_1^*(\delta_c) - p_1^{u*}(\delta_c) = \delta_c[\sigma_r^* - \sigma_n^*] - \delta_c[\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c))] = \delta_c[\sigma_r^* - \sigma_r^u(Q_1^{u*}(\delta_c))] + \delta_c[\sigma_n^u(Q_1^{u*}(\delta_c)) - \sigma_n^*] > 0$. Since $\partial_{p_2^u}(\mathbb{E}[(k + \alpha)V - p_2^u]^+ -$

$\mathbb{E}[(1+\alpha)V-p_2^u]^+) = \mathbb{P}[\frac{p_2^u}{1+k} \le V \le \frac{p_2^u}{k+\alpha}] > 0$ and $p_2^{r*} < p_2^u(X_2^n, X_2^r) < p_2^{n*}$ with probability 1, $\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)) < \mathbb{E}[(k+\alpha)V - p_2^{n*}]^+ - \mathbb{E}[(1+\alpha)V - p_2^{n*}]^+ = \mathbb{E}[(k+\alpha)V - p_2^{n*}]^+ - \sigma_n^*$. Hence, $p_1^*(\delta_c) - p_1^{u*}(\delta_c) > \delta_c[\sigma_r^* - \sigma_n^*] - \delta_c\{\mathbb{E}[(k+\alpha)V - p_2^{n*}]^+ - \sigma_n^*\} = \delta_c(\sigma_r^* - \mathbb{E}[(k+\alpha)V - p_2^{n*}]^+)$ for all $\delta_c > 0$. It is also straightforward to check that, for any $\delta_c \in [0,\delta]$ and $Q_1 > 0$, $\Pi_f(Q_1|\delta_c) > \Pi_f^u(Q_1|\delta_c)$ for all $Q_1 > 0$ and $\delta_c \in [0,\delta]$. Therefore, $\Pi_f^*(\delta_c) = \max_{Q_1} \Pi_f(Q_1|\delta_c) > \max_{Q_1} \Pi_f^u(Q_1|\delta_c) = \Pi_f^{u*}(\delta_c)$.

Part (c). Note that $\Pi_f(Q_1|\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}[Q_1 \wedge X] - (c_1 - s)Q_1 + \delta\beta_n^*\mathbb{E}[X]$. By the proof of Theorem 2(a), it is easy to check that, if $m_1^*(\delta_c) = \mu + \delta(\beta_r^* - \beta_n^*) + \delta_c(\sigma_r^* - \sigma_n^*)$ is increasing in k at $k = k_0$, it is increasing in k for all $k \le k_0$. In other words, $m_1^*(\delta_c)$ is quasiconcave in k. Furthermore, since c_1 is convexly decreasing in k, following the same argument as the proof of Theorem 2(a), direct computation yields that the critical fractile $\frac{c_1 - s}{m_1^*(\delta_c) - s}$ is decreasing in k at k_0, so it is decreasing in k for all $k \le k_0$. Thus, $\frac{c_1 - s}{m_1^*(\delta_c) - s}$ is quasiconvex in k, and, therefore, there exists a K such that $Q_1^*(\delta_c)$ is increasing in k if $k \le K$, and decreasing in k if $k \ge K$. Next we show that $\Pi_f^*(\delta_c)$ is also increasing in k when $k \le K$ and decreasing in k when $k \ge K$. It is clear that $K = \arg\min_k \left[\frac{c_1 - s}{m_1^*(\delta_c) - s}\right]$. Since c_1 is convexly decreasing in k, for any realization of X and production quantity $Q_1 = Q_1^*(\delta_c) = \bar{F}^{-1}\left(\frac{c_1 - s}{m_1^*(\delta_c) - s}\right)$, $(m_1^*(\delta_c) - s)(Q_1 \wedge X) - (c_1 - s)Q_1$ is increasing in k for $k \le K$, and decreasing in k for $k \ge K$. Therefore, $\Pi_f^*(\delta_c) = \mathbb{E}[(m_1^*(\delta_c) - s)(Q_1^*(\delta_c) \wedge X) - (c_1 - s)Q_1^*(\delta_c) + \delta\beta_n^* X]$ is increasing in k when $k \le K$, and it is decreasing in k when $k \ge K$.

Part (d). Since $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$, $|\partial_k p_1^*(\delta_c)| = \delta_c|\partial_k \sigma_r^*|$, which is clearly increasing in δ_c. *Q.E.D.*

Before showing Theorem 4, we first prove Theorem 5 and Theorem 6.

Proof of Theorem 5: Part (a). Since $Q_1^*(\cdot)$ and $Q_1^{u*}(\cdot)$ are continuous in δ_c, it suffices to show that $Q_1^*(\delta) > Q_1^{u*}(\delta)$. We first show that $m_1^u(Q_1|\delta)$ is decreasing in Q_1. Observe that $m_1^u(Q_1|\delta) = \mu + \delta[U_r(Q_1) - U_n(Q_1)]$, where $U_r(Q_1) := \mathbb{E}\left[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)\right] + \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+$, and $U_n(Q_1) := \mathbb{E}\left[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)\right] + \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+$. Let $u_r(p) := (p - c_2)\bar{G}(\frac{p}{k+\alpha}) + \mathbb{E}((k+\alpha)V - p)^+ = \mathbb{E}[(k+\alpha)V - c_2]1_{\{(k+\alpha)V \ge p\}}$ and $u_n(p) := (p - c_2)\bar{G}(\frac{p}{1+\alpha}) + \mathbb{E}((1+\alpha)V - p)^+ = \mathbb{E}[(1+\alpha)V - c_2]1_{\{(1+\alpha)V \ge p\}}$. It's clear that $u_r(\cdot)$ and $u_n(\cdot)$ are continuously decreasing in p. Moreover, $U_r(Q_1) = \mathbb{E}[u_r(p_2^u(X_2^n, X_2^r))]$ and $U_n(Q_1) = \mathbb{E}[u_n(p_2^u(X_2^n, X_2^r))]$, where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. Since $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r, it is stochastically decreasing in Q_1. Hence, it suffices to show that $u_r(p) - u_n(p)$ is increasing in p. Observe that $u_r(p) - u_n(p) = -[\int_{p/(1+\alpha)}^{\bar{v}}((1+\alpha)V - \max(p, (k+\alpha)V))g(V)\,\mathrm{d}V]$, which is continuously increasing in p. Therefore, $m_1^u(Q_1|\delta) = \mu + \delta(U_r(Q_1) - U_n(Q_1)) = \mu + \delta\{\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_r(p_2^u(X_2^n, X_2^r))]\}$ is continuously decreasing in Q_1.

We now show that $m_1^u(Q_1|\delta) < m_1^*(\delta)$ for all Q_1. Observe that $m_1^u(Q_1|\delta) - m_1^*(\delta) = \delta\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] - \delta\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})]$. Because $p_2^{r*} \le p_2^u(X_2^n, X_2^r) \le p_2^{n*}$ and $u_r(\cdot)$ and $u_n(\cdot)$ are decreasing in p, $\delta\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] \le 0$ and $\delta\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})] \ge 0$. Hence, $m_1^u(Q_1|\delta) \le m_1^*(\delta)$. Since $k < 1$, $p_2^{r*} < p_2^{n*}$, one of the inequalities $\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] \le 0$ and $\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})] \ge 0$ must be strict. Therefore, $m_1^u(Q_1|\delta) < m_1^*(\delta)$ for all $Q_1 \ge 0$.

Next, we show that $Q_1^*(\delta) > Q_1^{u*}(\delta)$. Observe that $\Pi_f^u(Q_1|\delta) - \Pi_f(Q_1|\delta) = (m_1^u(Q_1|\delta) - m_1^*(\delta))\mathbb{E}(X \wedge Q_1) + \delta\mathbb{E}\left[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) - \beta_n^*\right] X$. Let $\Pi(Q_1, 1) = \Pi_f(Q_1|\delta)$ and $\Pi(Q_1, 0) = \Pi_f^u(Q_1|\delta)$. Then, $\Pi(Q_1, 1) - \Pi(Q_1, 0) = (m_1^*(\delta) - m_1^u(Q_1|\delta))\mathbb{E}(X \wedge Q_1) + \delta\mathbb{E}[\beta_n^* - (p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)]X$. Since $m_1^*(\delta) \geq m_1^u(Q_1|\delta)$ and $m_1^u(Q_1|\delta)$ is decreasing in Q_1, $(m_1^*(\delta) - m_1^u(Q_1|\delta))\mathbb{E}(X \wedge Q_1)$ is increasing in Q_1. Also note that $p_2^u(X_2^n, X_2^r)$ and thus $(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)$ is decreasing in Q_1. Therefore, $\Pi(Q_1, 1) - \Pi(Q_1, 0)$ is increasing in Q_1. Hence, $\Pi(\cdot, \cdot)$ is supermodular on the lattice $[0, +\infty) \times \{0, 1\}$ and $Q_1^{u*}(\delta) = \arg\max_{Q_1 \geq 0} \Pi_f^u(Q_1|\delta) \leq \arg\max_{Q_1 \geq 0} \Pi_f(Q_1|\delta) = Q_1^*(\delta)$. Since $m_1^*(\delta) > m_1^u(Q_1^{u*}(\delta)|\delta)$, $\partial_{Q_1}\Pi_f(Q_1^{u*}(\delta)|\delta) > \partial_{Q_1}\Pi_f^u(Q_1^{u*}(\delta)|\delta) = 0$. Since $\Pi_f(\cdot|\delta)$ is concave in Q_1, $Q_1^*(\delta) > Q_1^{u*}(\delta)$. Due to the continuity of $Q_1^*(\cdot)$ and $Q_1^{u*}(\cdot)$ in δ_c, there exists a threshold $\bar{\delta}_q \leq \delta$ such that $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$ for all $\delta > \bar{\delta}_q$.

Part (b). We first show that $m_1^u(Q_1|0)$ is increasing in Q_1. Note that $m_1^u(Q_1|0) = \mu + \delta(\beta_r^u(Q_1) - \beta_n^u(Q_1))$. By Lemma 3, $\beta_r^u(\cdot)$ is increasing whereas $\beta_n^u(\cdot)$ is decreasing in Q_1. Therefore, $m_1^u(Q_1|0)$ is increasing in Q_1.

We then show that there exists a threshold $\bar{Q}_1$ such that $m_1^u(Q_1|0) > m_1^*(0)$ ($m_1^u(Q_1|0) < m_1^*(0)$) if $Q_1 > \bar{Q}_1$ ($Q_1 < \bar{Q}_1$). Let $\hat{\beta}_r^* = \max_{p \geq 0} \hat{v}_2^r(p) = \lim_{Q_1 \to +\infty} \beta_r^u(Q_1)$. Since $k < 1$, $\hat{\beta}_n^* := v_2^n(\hat{p}_2^{r*}) < \beta_n^*$. It is clear that $\beta_r^* - \hat{\beta}_r^*$ is increasing in r_2, with $\beta_r^* = \hat{\beta}_r^*$ if $r_2 = 0$. Let $\bar{r}_2 > 0$ be the threshold such that $\beta_r^* - \hat{\beta}_r^* = \beta_n^* - \hat{\beta}_n^*$. Hence, $\beta_r^* - \hat{\beta}_r^* < \beta_n^* - \hat{\beta}_n^*$ for all $r_2 < \bar{r}_2$. Moreover, by the monotone convergence theorem, $\lim_{Q_1 \to +\infty} m_1^u(Q_1|0) = \mu + \delta[v_2^r(\hat{p}_2^{r*}) - v_2^n(\hat{p}_2^{r*})] = \mu + \delta[\hat{\beta}_r^* - \hat{\beta}_n^*] > \mu + \delta[\beta_r^* - \beta_n^*] = m_1^*(0)$. Since $m_1^u(Q_1|0)$ is increasing in Q_1, there exists a threshold $\bar{Q}_1$ such that $m_1^u(Q_1|0) > m_1^*(0)$ ($m_1^u(Q_1|0) < m_1^*(0)$) if $Q_1 > \bar{Q}_1$ ($Q_1 < \bar{Q}_1$).

Now we show there exists a $c_q > 0$ such that, if $c_1 < c_q$, $Q_1^*(0) < Q_1^{u*}(0)$. It is clear that $Q_1^{u*}(0) \uparrow \bar{X}$ and $Q_1^*(0) \uparrow \bar{X}$ as $c_1 \downarrow 0$, where $\bar{X}$ is the upper bound of the support of X ($\bar{X}$ may take the value of $+\infty$). Hence, there exists a threshold $c_q > 0$ (dependent on r_2) such that if $c_1 < c_q$, $Q_1^{u*}(0) > \bar{Q}_1$ and $Q_1^*(0) > \bar{Q}_1$. Let $\hat{\pi}_2(Q_1) := \delta\mathbb{E}[v_2^n(p_2^u(X_2^n, X_2^r))X]$, where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. It's clear that $\hat{\pi}_2(\cdot)$ is differentiable and, by the chain rule $\hat{\pi}_2'(Q_1) = \delta\mathbb{E}[\partial_p v_2^n(p_2^u(X_2^n, X_2^r))(\partial_{X_2^n} p_2^u(X_2^n, X_2^r) + \partial_{X_2^r} p_2^u(X_2^n, X_2^r))1_{\{X \geq Q_1\}}X]$. As $Q_1 \to \bar{X}$, for any realization of $X \leq \bar{X}$, $\partial_{X_2^n} p_2^u(X_2^n, X_2^r)$ and $\partial_{X_2^r} p_2^u(X_2^n, X_2^r)$ converges to 0. Hence, by the dominated convergence theorem, there exits a threshold $\hat{Q} \in [\bar{Q}_1, \bar{X})$, such that $\hat{\pi}_2'(Q_1) \in [-\epsilon\mathbb{P}(X \geq Q_1), 0]$ for all $Q_1 \geq \hat{Q}$, where $\epsilon := (\tilde{m}_1^u(\hat{Q}) - \tilde{m}_1^*)/2 > 0$. Let $\bar{c}_1(r_2) \in (0, \tilde{c}(r_2)]$ be the threshold such that, if $c_1 < \bar{c}_1(r_1)$, we have $Q_1^{u*}, Q_1^* > \hat{Q} \geq \bar{Q}_1$. Therefore, $\partial_{Q_1}\Pi_f(Q_1^{u*}(0)|0) = (m_1^*(0) - r_1)\mathbb{P}(X \geq Q_1^{u*}(0)) - (c_1 - r_1) < (m_1^u(Q_1^{u*}(0)|0) - r_1)\mathbb{P}(X \geq Q_1^{u*}(0)) - \epsilon\mathbb{P}(X \geq Q_1^{u*}(0)) - (c_1 - r_1) \leq (m_1^u(Q_1^{u*}(0)|0) - r_1)\mathbb{P}(X \geq Q_1^{u*}(0)) + \hat{\pi}_2'(Q_1^{u*}(0)) - (c_1 - r_1) \leq \partial_{Q_1}\Pi_f^u(Q_1^{u*}(0)|0) = 0$, where the first inequality follows from $m_1^u(Q_1^{u*}(0)|0) - m_1^*(0) \geq (m_1^u(\hat{Q}|0) - m_1^*(0)) = 2\epsilon > \epsilon$, the second from $\hat{\pi}_2'(Q_1^{u*}(0)) \in [-\epsilon\mathbb{P}(X \geq Q_1^{u*}(0)), 0]$, and the last from the monotonicity that $m_1^u(\cdot|0)$ is increasing in Q_1. Because $\Pi_f(\cdot|0)$ is concave in Q_1, $Q_1^*(0) = \arg\max_{Q_1} \Pi_f(Q_1|0) < Q_1^{u*}(0)$ follows immediately. Since $Q_1^*(\delta_c)$ and $Q_1^{u*}(\delta_c)$ are continuous in δ_c, there exists a threshold $\underline{\delta}_q$ such that $Q_1^*(\delta_c) < Q_1^{u*}(\delta_c)$ for all $\delta_c \in [0, \underline{\delta}_q)$.

Part (c). By Theorem 2, $Q_1^*(\delta_c)$ is strictly increasing in δ_c if $\sigma_r^* > \sigma_n^*$, whereas $Q_1^{u*}(\delta_c)$ is strictly decreasing in δ_c. Therefore, $\underline{\delta}_q = \bar{\delta}_q$ if $\sigma_r^* > \sigma_n^*$. *Q.E.D.*

Proof of Theorem 6: Part(a). A straightforward algebraic manipulation yields $I_e^*(\delta_c) = I_e(Q_1^*(\delta_c))$, where $I_e(Q_1) := \kappa_1 Q_1 + \left[\delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) - \delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{1+\alpha}\right) - \delta\iota_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{E}(X \wedge Q_1) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}[X]$. If $\kappa_1 \geq \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$, it is easy to check that $I_e'(Q_1^*(\delta_c)) > \left[\kappa_1 - \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{P}(X \geq$

$Q_1^*(\delta_c)) > 0$ where the first inequality follows from $\mathbb{P}(X \geq Q_1^*(\delta_c)) < 1$, whereas the second inequality follows from the assumptions that $\kappa_1 \geq \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$ and $\kappa_2 > \iota_2$. Thus, by Theorem 2(a), if $\sigma_r^* > \sigma_n^*$, $Q_1^*(\delta_c)$ is strictly increasing in δ_c, so is $I_e^*(\delta_c) = I_e(Q_1^*(\delta_c))$; if $\sigma_r^* < \sigma_n^*$, $Q_1^*(\delta_c)$ is strictly decreasing in δ_c, so is $I_e^*(\delta_c) = I_e(Q_1^*(\delta_c))$. Furthermore, by Theorem 2, $\sigma_r^* > \sigma_n^*$ if and only if $k \in (\underline{k}, \bar{k})$, and $\sigma_r^* < \sigma_n^*$ if and only if $k < \underline{k}$ or $k > \bar{k}$.

Part (b). As shown in part(a), $I_e^*(\delta_c)$ is strictly increasing in $Q_1^*(\delta_c)$ and, by the proof of Theorem 3(c), $Q_1^*(\delta_c)$ is increasing in k when k is small, and decreasing in k when k is big. Therefore, with the same argument as Theorem 3, we know $I_e^*(\delta_c)$ is quasiconcave in k, and thus there exists a threshold K_e, such that $I_e^*(\delta_c)$ is increasing (resp. decreasing) in k for $k \leq K_e$ (resp. $k \geq K_e$). *Q.E.D.*

Proof of Theorem 4: Part (a). Since $\delta_c > \bar{\bar{\delta}}_q$, Theorem 5(a) implies that $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$. Now we compute $I_e^{u*}(\delta_c)$. Given the market size (X_2^n, X_2^r), the equilibrium total second-period production quantity, $Q_2^u(X_2^n, X_2^r)$, is given by $Q_2^u(X_2^n, X_2^r) = X_2^n\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) + X_2^r\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)$. Therefore, following the same argument as in the proof of Theorem 6, we have $I_e^{u*}(\delta_c) = \mathbb{E}\{\kappa_1 Q_1^{u*}(\delta_c) + \delta\kappa_2 Q_2^u(X_2^{n*}, X_2^{r*})\} = \kappa_1 Q_1^{u*}(\delta_c) + \mathbb{E}\left[\left(\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right) - \delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)\right)(X \wedge Q_1^{u*}(\delta_c))\right] + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)X\right]$, where $X_2^{n*} = (X - Q_1^{u*}(\delta_c))^+$ and $X_2^{r*} = X \wedge Q_1^{u*}(\delta_c)$. For any δ_c, $I_e^*(\delta_c)$ is strictly linearly decreasing in ι_2. Thus, let $\bar{\iota}_e := \max\{\iota_2 : I_e^*(\delta_c) \geq I_e^{u*}(\delta_c)\}$. We have $I_e^*(\delta_c) > I_2^{u*}(\delta_c)$, if $\iota_2 < \bar{\iota}_e$. In particular, if $\iota_2 = 0$, $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$, $p_2^{r*} < p_2^u(\cdot,\cdot) < p_2^{n*}$, and $\kappa_1 \geq \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$ imply that $\kappa_1 Q_1^*(\delta_c) - \left[\delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\right]\mathbb{E}(X \wedge Q_1^*(\delta_c)) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}[X] > \kappa_1 Q_1^{u*}(\delta_c) - \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)\right](X \wedge Q_1^{u*}(\delta_c)) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)X\right]$, and $\delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\mathbb{E}(X \wedge Q_1^*(\delta_c)) > \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right)(X \wedge Q_1^{u*}(\delta_c))\right]$. Thus, for $\iota_2 = 0$, we follow the same argument as the proof of Theorem 6 to establish that $I_e^*(\delta_c) = \kappa_1 Q_1^*(\delta_c) - \left[\delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\right]\mathbb{E}(X \wedge Q_1^*(\delta_c)) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}[X] + \delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\mathbb{E}(X \wedge Q_1^*(\delta_c)) > \kappa_1 Q_1^{u*}(\delta_c) - \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)\right](X \wedge Q_1^{u*}(\delta_c)) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)X\right] + \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right)\mathbb{E}(X \wedge Q_1^{u*}(\delta_c))\right] = I_e^{u*}(\delta_c)$, i.e., $I_e^*(\delta_c) > I_e^{u*}(\delta_c)$ for $\iota_2 = 0$. Therefore, $\bar{\iota}_e > 0$.

Part (b). Since $\delta_c < \underline{\delta}_q$, Theorem 5(b) implies that $Q_1^*(\delta_c) < Q_1^{u*}(\delta_c)$. Lemma 2 implies that $p_2^{r*} < p_2^u(\cdot,\cdot) < p_2^{n*}$. Hence, $\kappa_1 Q_1^*(\delta_c) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\{\mathbb{E}[X] - \mathbb{E}(X - Q_1^*(\delta_c))^+\} < \kappa_1 Q_1^{u*}(\delta_c) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)X - (X \wedge Q_1^{u*}(\delta_c))^+\right]$. Let $\tilde{\iota}_e := (\bar{G}(\frac{p_2^{r*}}{k+\alpha}) - \bar{G}(\frac{p_2^{n*}}{k+\alpha}))\kappa_2/\bar{G}(\frac{p_2^{r*}}{k+\alpha}) < \kappa_2$. If $\iota_2 > \tilde{\iota}_e$, since $Q_1^{u*}(\delta_c) > Q_1^*(\delta_c)$ and $p_2^{r*} < p_2^u(\cdot,\cdot) < p_2^{n*}$, $\mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right)(Q_1^{u*}(\delta_c) \wedge X)\right] > \left[\delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{E}(Q_1^{u*}(\delta_c) \wedge X) > \left[\delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{E}(Q_1^*(\delta_c) \wedge X)$. Putting everything together, if $\iota_2 > \tilde{\iota}_e$, we have that $I_e^{u*}(\delta_c) = \kappa_1 Q_1^{u*}(\delta_c) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)X - (X \wedge Q_1^{u*}(\delta_c))^+\right] + \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right)\right](Q_1^{u*}(\delta_c) \wedge X) > \kappa_1 Q_1^*(\delta_c) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\{\mathbb{E}[X] - \mathbb{E}(X - Q_1^*(\delta_c))^+\} + \mathbb{E}\{\left[\delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right](Q_1^*(\delta_c) \wedge X)\} = I_e^*(\delta_c)$. This shows part (b). *Q.E.D.*

Proof of Theorem 7: We first derive $S_c^*(\delta_c)$ and $S_c^{u*}(\delta_c)$. Let $\mathfrak{a}_1^*(\delta_c)$ and $\mathfrak{a}_1^{u*}(\delta_c)$ be the in-stock probability in the base model and the NTR model, respectively. The expected surplus of a customer with discount factor δ_c in the base model is given by: $\mathfrak{a}_1^*(\delta_c)(\mu - p_1^*(\delta_c) + \delta\sigma_r^*) + (1 - \mathfrak{a}_1^*(\delta_c))\delta\sigma_n^* = \mathfrak{a}_1^*(\delta_c)(\mu - \mu - \delta_c(\sigma_r^* - \sigma_n^*) + \delta\sigma_r^*) + (1 - \mathfrak{a}_1^*(\delta_c))\delta\sigma_n^* = \mathfrak{a}_1^*(\delta_c)(\delta - \delta_c)(\sigma_r^* - \sigma_n^*) + \delta\sigma_n^*$, where the first equality follows from $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$. Therefore, the equilibrium total customer surplus is given by

$S_c^*(\delta_c) = \mathbb{E}[(\mathfrak{a}_1^*(\delta_c)(\delta - \delta_c)(\sigma_r^* - \sigma_n^*) + \delta\sigma_n^*)X]$. Analogously, the expected surplus of a customer with discount factor δ_c in the NTR model is given by: $\mathfrak{a}_u^*(\delta_c)(\mu - p_1^{u*}(\delta_c) + \delta\sigma_r^u(Q_1^{u*}(\delta_c))) + (1 - \mathfrak{a}_u^*(\delta_c))\delta\sigma_n^u(Q_1^{u*}(\delta_c)) = \mathfrak{a}_u^*(\delta_c)(\delta - \delta_c)(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c))) + \delta\sigma_n^u(Q_1^{u*}(\delta_c))$. Therefore, the equilibrium total customer surplus is given by $S_c^{u*}(\delta_c) = \mathbb{E}[(\mathfrak{a}_u^*(\delta_c)(\delta - \delta_c)(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c))) + \delta\sigma_n^u(Q_1^{u*}(\delta_c)))X]$.

Next, we show that $S_c^*(\delta_c) < S_c^{u*}(\delta_c)$ for $\delta_c > \bar{\delta}_s$. Note that, when $\delta_c = \delta$, $S_c^*(\delta_c) = \delta\mathbb{E}[\sigma_n^* X]$ and $S_c^{u*}(\delta_c) = \delta\mathbb{E}[\sigma_n^u(Q_1^{u*}(\delta_c))X]$. By Lemma 2(c), $\sigma_n^* < \sigma_n^u(Q_1^{u*}(\delta_c))$. Hence, it follows immediately that $S_c^{u*}(\delta_c) = \delta\mathbb{E}[\sigma_n^u(Q_1^{u*}(\delta_c))X] > \delta\mathbb{E}[\sigma_n^* X] = S_c^*(\delta_c)$ for $\delta_c = \delta$. Since $S_c^*(\delta_c)$ and $S_c^{u*}(\delta_c)$ are continuous in δ_c, there exists a threshold $\tilde{\delta}_s < \delta$ such that $S_c^{u*}(\delta_c) > S_c^*(\delta_c)$ for $\delta_c \in (\tilde{\delta}_s, \delta]$. *Q.E.D.*

Proof of Theorem 8: Part (a). It follows from the same argument as the proof of Theorem 1(a) that, $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$. Let $W_s(Q_1|\delta_c)$ denote the expected total social welfare with first-period production quantity Q_1 and customer discount factor δ_c. To compute $W_s(Q_1|\delta_c)$, Since $w_2(X_2^n, X_2^r) = \sigma_n^{s*}X_2^n + \sigma_r^{s*}X_2^r$, we have $W_s(Q_1|\delta_c) = p_1^{s*}(\delta_c)\mathbb{E}(X \wedge Q_1) + (\mu - p_1^{s*}(\delta_c))\mathbb{E}(X \wedge Q_1) - (c_1 + \kappa_1)Q_1 + s\mathbb{E}(Q_1 - X)^+ + \delta\mathbb{E}\{w_2(X - (X \wedge Q_1), X \wedge Q_1)\} = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s + \kappa_1)Q_1 + \delta\sigma_n^{s*}\mathbb{E}(X)$. Therefore, $Q_1^{s*}(\delta_c)$ is the solution to a newsvendor problem with marginal revenue $m_1^{s*} - s$, marginal cost $c_1 + \kappa_1 - s$, and demand distribution $F(\cdot)$. Hence, $Q_1^{s*}(\delta_c) = \bar{F}^{-1}(\frac{c_1 + \kappa_1 - s}{m_1^{s*} - s})$, and the equilibrium social welfare is $W_s^*(\delta_c) = W_s(Q_1^{s*}(\delta_c)|\delta_c) = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 + \kappa_1 - s)Q_1^{s*}(\delta_c) + \delta\sigma_n^{s*}\mathbb{E}(X)$.

Part (b). It follows immediately from part (a) that $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$ is strictly increasing (resp. decreasing) in δ_c if and only if $\sigma_r^{s*} > \sigma_n^{s*}$ (resp. $\sigma_r^{s*} < \sigma_n^{s*}$). Note that, by part (a), $\sigma_r^{s*} = \mathbb{E}[(k + \alpha)V_2 - c_2 - \kappa_2 + e_2]^+$, where $e_2 := r_2 + \iota_2$. The same argument as the proof of Theorem 2(a) implies that if σ_r^{s*} is increasing in k at $k = k_0$, then σ_r^{s*} is increasing in k for all $k \le k_0$. Hence, σ_r^{s*} is quasiconcave in k. Let $\underline{k}_s := \arg\max_k \sigma_r^{s*} > \sigma_n^{s*}$ and $\bar{k}_s := \arg\max_k \sigma_r^{s*} > \sigma_n^{s*}$. The quasiconcavity of σ_r^{s*} in k suggests that $\sigma_r^{s*} > \sigma_n^{s*}$ if and only if $k \in (\underline{k}_s, \bar{k}_s)$, and $\sigma_r^{s*} < \sigma_n^{s*}$ if and only if $k < \underline{k}_s$ or $k > \bar{k}_s$. Since m_1^{s*} is independent of δ_c, $Q_1^{s*}(\delta_c)$ is independent of δ_c as well. As a result, $W_s^*(\delta_c) = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 + \kappa_1 - s)Q_1^{s*}(\delta_c) + \delta\sigma_n^{s*}\mathbb{E}(X)$ is independent of δ_c.

Part (c). The same argument as the proof of Theorem 3(c) demonstrates that $\frac{c_1 + \kappa_1 - s}{m_1^{s*} - s}$ is quasiconvex in k. Let $K_s := \arg\min_k \left[\frac{c_1 + \kappa_1 - s}{m_1^{s*} - s}\right]$. We have Q_1^{s*} is increasing in k for $k \le K_s$ and decreasing in k otherwise. Since c_1 is convexly decreasing in k, for any realization of X and production quantity $Q_1 = Q_1^{s*} = \bar{F}^{-1}\left(\frac{c_1 + \kappa_1 - s}{m_1^{s*} - s}\right)$, $(m_1^{s*} - s)(Q_1 \wedge X) - (c_1 + \kappa_1 - s)Q_1$ is increasing in k for $k \le K_s$ and decreasing in k otherwise. Therefore, $W_s^* = \mathbb{E}[(m_1^{s*} - s)(Q_1^{s*} \wedge X) - (c_1 + \kappa_1 - s)Q_1^{s*} + \delta\sigma_n^{s*}X]$ is also increasing in k if $k \le K_s$ and decreasing in k otherwise. *Q.E.D.*

Proof of Theorem 9: If $s_2^*(\delta_c)$ is the solution to $p_s^{n*} = \arg\max_{p_2^n \ge 0}(p_2^n + s_2 - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$, it is clear that the subsidy/tax scheme with $s_2 = s_2^*(\delta_c)$ can induce the equilibrium price p_s^{n*} for new customers. We now show that $s_2^*(\delta_c)$ exists. Since $v_2^n(p_2^n)$ is quasiconcave in p_2^n for any s_2, the first-order condition $\partial_{p_2^n}v_2^n(p_2^n) = 0$ guarantees the optimal price for new customers. Moreover, $\partial_{p_2^n}v_2^n(p_s^{n*}) = \bar{G}\left(\frac{p_s^{n*}}{1+\alpha}\right) - \frac{p_s^{n*} + s_2 - c_2}{1+\alpha}g\left(\frac{p_s^{n*}}{1+\alpha}\right)$, which is strictly decreasing in s_2. Hence, there exists a unique $s_2^*(\delta_c)$, such that $\partial_{p_2^n}v_2^n(p_s^{n*}) = 0$, thus inducing the socially optimal equilibrium price p_s^{n*} for new customers.

If $s_r^*(\delta_c)$ is the solution to $p_s^{r*} = \arg\max_{p_2^r \geq 0}(p_2^r + s_2^*(\delta_c) + s_r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right)$, the subsidy/tax scheme with $s_r = s_r^*(\delta_c)$ can induce the equilibrium trade-in price p_s^{r*} for repeat customers. We now show that $s_r^*(\delta_c)$ exists. Since $v_2^r(p_2^r)$ is quasiconcave in p_2^r for any (s_2, s_r), the first-order condition $\partial_{p_2^r} v_2^r(p_2^r) = 0$ guarantees the optimal price for new customers. Moreover, if $s_2 = s_2^*(\delta_c)$, $\partial_{p_2^r} v_2^r(p_s^{r*}) = \bar{G}\left(\frac{p_s^{r*}}{k+\alpha}\right) - \frac{p_s^{r*} + s_2^*(\delta_c) + s_r - c_2 + r_2}{k+\alpha} g\left(\frac{p_s^{r*}}{k+\alpha}\right)$, which is strictly decreasing in s_r. Hence, there exists a unique $s_r^*(\delta_c)$, such that $\partial_{p_2^r} v_2^r(p_s^{r*}) = 0$, thus inducing the socially optimal equilibrium trade-in price for repeat customers p_s^{r*}.

Given the subsidy/tax scheme $(s_1, s_2^*(\delta_c), s_r^*(\delta_c))$, as shown above, the firm adopts the same second-period pricing strategy as the social welfare maximizing one: (p_s^{n*}, p_s^{r*}). Hence, the first-period price should also be the same as the one that is socially optimal: $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$. Thus, the expected profit of the firm in period 1 is $\Pi_f^s(Q_1|\delta_c) = (m_1^s(s_1|\delta_c) - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s)Q_1 + \delta(p_s^{n*} + s_2^*(\delta_c) - c_2)\bar{G}\left(\frac{p_s^{n*}}{1+\alpha}\right)\mathbb{E}(X)$, where $m_1^s(s_1|\delta_c) = p_1^{s*}(\delta_c) + \delta[(\kappa_2 + s_2^*(\delta_c) + s_r^*(\delta_c) - \iota_2)\bar{G}(\frac{p_s^{r*}}{k+\alpha}) - (\kappa_2 + s_2^*(\delta_c))\bar{G}(\frac{p_s^{n*}}{1+\alpha})] + s_1$. Thus, $\Pi_f^s(Q_1|\delta_c)$ has a unique optimizer $\bar{F}^{-1}(\frac{c_1 - s}{m_1^s(s_1|\delta_c) - s})$. Moreover, as shown in Theorem 8, $Q_1^{s*}(\delta_c) = \bar{F}^{-1}(\frac{c_1 + \kappa_1 - s}{m_1^{s*} - s})$. Therefore, if $s_1^*(\delta_c)$ is the unique solution to $\frac{c_1 - s}{m_1^s(s_1|\delta_c) - s} = \frac{c_1 + \kappa_1 - s}{m_1^{s*} - s}$, the optimal production quantity with the linear subsidy/tax scheme $s_g^*(\delta_c) = (s_1^*(\delta_c), s_2^*(\delta_c), s_r^*(\delta_c))$ is $Q_1^{s*}(\delta_c)$, which is the socially optimal first-period production quantity. *Q.E.D.*

Proof of Theorem 10: Part (a). Under the optimal subsidy/tax policy $s_g^*(\delta_c)$, the firm's profit is $\Pi_f^{s*}(\delta_c) = (p_1^{s*}(\delta_c) + s_1^*(\delta_c) - s)\mathbb{E}(Q_1^{s*}(\delta_c) \wedge X) - (c_1 - s)Q_1^{s*}(\delta_c) + \delta\mathbb{E}(X \wedge Q_1^{s*}(\delta_c))(k+\alpha)J\left(\frac{c_2 + \kappa_2 - e_2}{k+\alpha}\right) + \delta\mathbb{E}(X - Q_1^{s*}(\delta_c))^+(1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right) = \left(\frac{c_1 - s}{c_1 + \kappa_1 - s}(m_1^{s*} - s)\right)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 - s)Q_1^{s*}(\delta_c) + \delta\mathbb{E}[X](1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right)$, where we plug in $s_1^*(\delta_c) = \frac{c_1 - s}{c_1 + \kappa_1 - s}(m_1^{s*} - s) + s - \mu - \delta_c(\sigma_s^{r*} - \sigma_s^{n*}) + \delta(k+\alpha)J\left(\frac{c_2+\kappa_2-e_2}{k+\alpha}\right) - \delta(1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right)$, with $J(x) := \bar{G}(x)/h(x)$. It follows immediately from its formula expression that $\Pi_f^{s*}(\delta_c)$ is independent of δ_c.

Part (b). It's clear that $\delta\mathbb{E}[X](1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right)$ is independent of the depreciation factor k, whereas $\left(\frac{c_1 - s}{c_1+\kappa_1 - s}(m_1^{s*} - s)\right)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 - s)Q_1^{s*}(\delta_c)$ is a constant proportion of the optimal social welfare that is influenced by the production decision (i.e., $(m_1^{s*} - s)\mathbb{E}[X \wedge Q_1^{s*}(\delta_c)] - (c_1 - s + \kappa_1)Q_1^{s*}(\delta_c)$). By Theorem 8(c), $(m_1^{s*} - s)\mathbb{E}[X \wedge Q_1^{s*}(\delta_c)] - (c_1 - s + \kappa_1)Q_1^{s*}(\delta_c)$ is increasing in k for $k \leq K_s$ and decreasing in k for $k \geq K_s$, i.e., W_s^* is maximized at $k = K_s$. Therefore, the firm's profit under the subsidy/tax scheme $s_g^*(\delta_c)$, $\Pi_f^{s*}(\delta_c)$, is maximized at $k = K_s$ as well. In other words, if the firm has the flexibility to control the depreciation factor k (equivalently, the remanufacturing efficiency), it will set the socially optimal one K_s under the optimal government policy $s_g^*(\delta_c)$. *Q.E.D.*