Recent Development in Health-Care Operations

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Abstract

Health-care systems are now facing a great many of challenges in most countries. Some suffer from drastically high health-care expenditures while others are facing the challenges of inefficiency and inequity. Under such circumstances, operations research/operations management (OR/OM) scientists contribute significantly in developing analytical models that highlight trade-offs and help government, health-care institution managers, physicians and patients identify optimal solutions and inform decision making when resources are limited. This paper surveys the papers in the frontiers of health-care operations which mainly reflect the issues of, say, health-care policy, health-care OM and treatment optimization. We review the papers from the perspective of presenting problems, drawing a complete picture of the relevant problems that are accessible with OR/OM methods and discussing how OR/OM methods should be applied to analyze these health-care issues. The health-care problems are categorized into three groups: *policy*, *institution*, and *individual*, according to their scales. In each category, we present problem settings and review how analytical models and methods lead us to a comprehensive understanding of and easy-to-implement solution to these problems. Papers addressing issues in the *policy* category are devoted to analyze the efficiency, social influence, and overall costs of governmental, such as epidemics control and vaccine distribution, health-care policies. *Institution* papers mainly discuss issues faced by health-care institutions, say, hospitals and clinics. They make attempt improve their efficiency, with limited resources, in medical device management, appointment response and operating room scheduling. In the *individual* section, we discuss papers that address treatment optimization from individual patient's perspective. Both treatment and diagnosis are discussed in this section, with particular focus on radiation therapy and chemotherapy, as well as screening policy. We conclude our paper with some thoughts on future research directions.

Key words: Literature review; operations research; operations management; health-care systems

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1 Introduction

As is widely recognized, health-care systems in most countries are facing great challenges of, to name a few, astronomical costs, low quality, inconsistency and inefficiency. An increasing proportion of research in the area of OR/OM is devoted to developing models capable of analyzing heal-care issues quantitatively so as to improve heal-care system performance. It is the purpose of this paper to review this literature. Inspired by Romeijn et al. (2008) that divided issues in health-care operations into three categories: health-care policy, treatment optimization and health-care operations management, we review the literature in accordance with the scale of the issue it addresses.

The first part of the paper focuses on the papers that have contributed to our understanding of optimizing governmental and public medical policies. They mainly aim to to solve macroscopic problems that involve the action of a national organization or government. We first consider the dynamics of epidemics and government's optimal response to it. The next is vaccination problems, which deals with the selection, production and composition of vaccines applied to pediatric immunization and epidemic prevention. We then proceeds to discuss papers that study the optimal allocation of donated organs, which requires careful analysis of patient's choice. The final part of this section is contributed to a few other papers not included in the groups listed above.

The paper then makes some effort to discuss the literature that addresses the issues faced by health-care institutions. The majority of the health-care operations papers try to handle problems of this kind. Lying in the middle of the health-care system, health-care institutions, e.g. hospitals and clinics, must take into considerations the perspectives of both policy makers and patients. This is the reason there is ample literature in this area. Here we try to discuss the capacity management first. The goal is to understand how to allocate limited resources, like operating rooms and diagnostic devices, in a hospital or clinic, efficiently. We particularly emphasize the allocation of operating rooms. We then review papers conducting optimal appointment scheduling, which mainly deals with no-show and last-minute cancellations. The third part of this section contributes to a brief review of papers that embed supply chain management into health-care operations issues. Both reimbursement policy and models with vaccine yield uncertainty are emphasized here. We conclude this section with a few other topics at institution level, like, to name one, the revenue management for non-profit operations.

The third part of this paper then showcases papers that analyze treatment optimization problem from the patient's perspective. We review papers that inform patients or physicians the optimal treatment and diagnosis method for a disease, e.g. HIV or breast cancer, or a certain kind of therapy, radiation therapy or chemotherapy. This section is divided into two parts: diagnosis and treatment. For diagnosis, part of the literature contributes to our knowledge of applying statistical methods to connect symptoms with disease and computing methods to simulate the homeostasis of body water regulation to track disease. We also discuss the optimal screening policy of breast cancer so that to detect the disease before it is outwardly observable. For treatment, we present papers that analyze the optimal treatment initiation policy against some chronic disease like HIV and end-stage renal failure. The timing policy of this kind usually face the trade-off between the side-effects of the treatment and the risk of the possibility of irreversible damage. The end part of this section concentrates on radiation therapy and chemotherapy which are 2 most common approaches against cancers. To optimize these therapies, we need to both improve their efficacy of killing cancer cells and control their side effects of overdosing on nearby healthy tissues. A new method in radiation, intensity modulated radiation therapy (IMRT) that decomposes the beam into several beam-lets whose direction and intensity can

be controlled individually, is extensively reviewed.

The paper ends with a few thoughts on future research directions.

We put a few words on how we choose the papers and what we are going to do with them. The emphasis is modeling, not analysis so we present papers with modeling novelties. Within a collection of papers, if there is no clear logical progression, we will simply review them in chronological sequence. The mentioning of results is often brief and is meant to arouse your curiosity. We sometimes purposely specialize a model for ease of presentation and for crystallizing the main ideas without getting trapped in details.

2 Health-Care Policy

The perspective taken by this part of the literature is often that of a policy maker, who determines decision rules to optimize the performance of a health-care policy at the entire society level. The decision rules reflect the interest of the whole society who follows the rules. For example, it is the government's responsibility to respond to an influenza pandemic by optimizing the trade off between the vaccination cost and the cost due to the infected population. This section reviews papers that try to quantify the health-care policy issues at government level.

2.1 Response to Epidemics

A significant part of the literature is interested in understanding epidemic dynamics and the optimized governmental policy to dampen the explosion of an epidemic disease. We first consider papers that models different types of epidemics transmitted through contact. We then review atmospheric release models where the disease is transmitted through air with wind. This type of model is usually applied to the bio-terrorism case where the government needs to respond to anthrax or smallpox attack. Coordination and competition are widely considered in almost all subareas of OR/OM, so are they in the response to epidemics. Papers focusing on the coordination and competition are considered at the end of this subsection.

We present the papers addressing problems involved with modeling epidemic dynamics so as to project the number of patients at a given time. This is the most common and most widely studied topic in epidemiology. A great many mathematical models are developed to understand the dynamics of such diseases and the efficient methods control them. You may refer to Anderson et al. (1991) for a complete picture of this issue. We mainly review a few recent papers that reveals modeling novelty.

Contact-transmitted epidemics

Zaric et al. (2000) studies HIV prevention by considering its connection with the methadone maintenance treatment to heroin addiction. The analysis is based on a dynamic compartmental model of the HIV epidemic among a population of adults, ages 18 to 44. The population is divided into nine compartments according to infection status and risk group. Each compartment is characterized by whether the population is not-infected, infected with HIV or infected with AIDS and by whether it is injection drug user (IDU) with methadone maintenance treatment (MMT), IDU without MMT or non-IDU. The model takes into account disease transmission from drug injection and sexual contacts. The health benefits of methadone maintenance and the resulting HIV infections averted are measured in terms of life years gained and quality-adjusted life years gained. Costs considered include health-care costs and the cost of methadone maintenance. The dynamics of HIV epidemic is represented by a

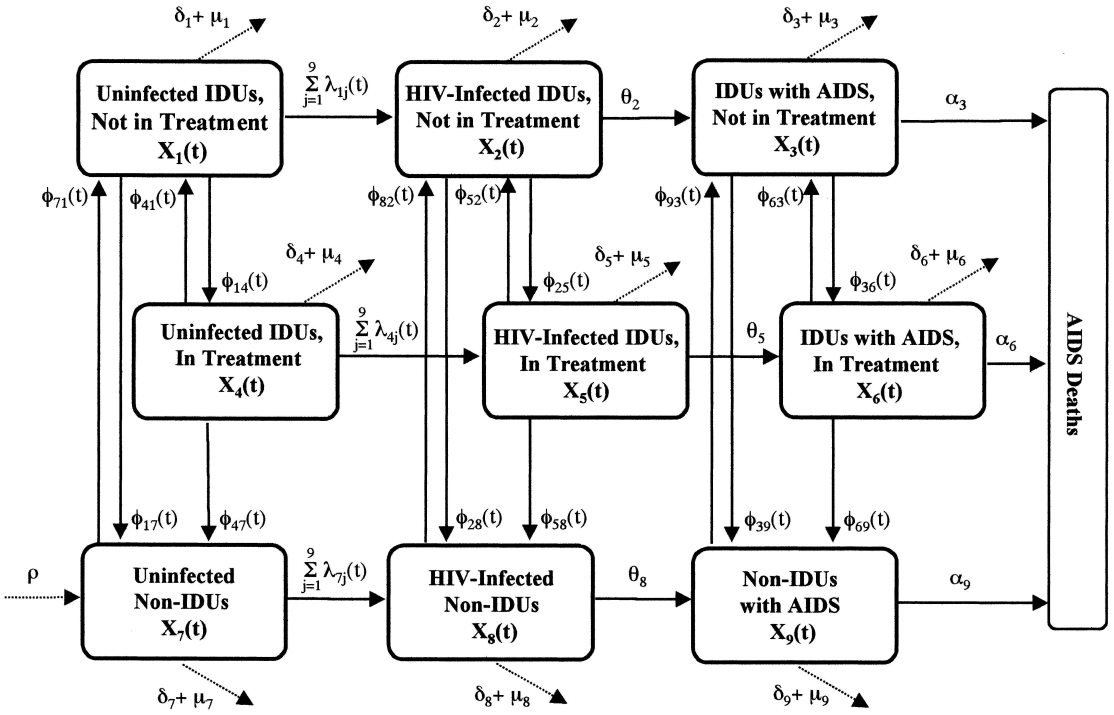
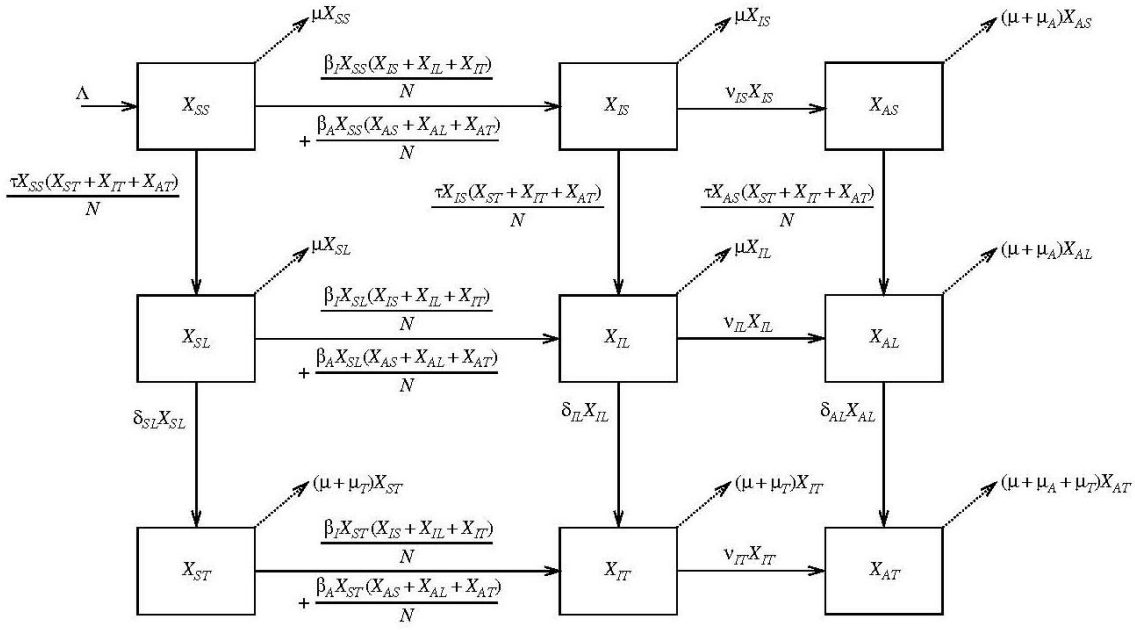


Figure 1: Schematic of Compartmental Model of HIV Epidemic and MMT

system of 9 non-linear differential equations that project the number of individuals in compartment i at time t. Figure 1 illustrates the dynamics with a single scheme. The population in each compartment is transfered to each other through getting into and out of treatment and addiction, maturation, non-AIDS and AIDS death, as well as HIV infection caused by sexual and injection-needle contact.

The paper investigates the cost-effectiveness of expanding MMT by 10% and shows, by numerical experiment, that the expansion is cost-effective. Consideration of quality of life makes expansion of current MMT programs appear to be more cost-effective than when only life years are considered. This occurs because the increase in treatment capacity not only increases the number of life years to live, but also increases their quality. The majority of the cost savings generated by the additional treatment slots are savings in HIV-care costs. Methadone maintenance reduces risky needle-sharing and sexual behavior among treated IDUs, and this reduces the risk of HIV infection for IDUs and for sex partners of IDUs.

HIV prevention and treatment is always a hot spot of health-care research. Long et al. (2008) makes some effort to understand the co-epidemics of HIV and tuberculosis(TB) and shows that exclusively treating HIV or TB reduces the targeted epidemic but exacerbates the other epidemic subsequently. We now consider a model in which one disease has three states (susceptible, infected with no symptoms, and infected with symptoms), and the second disease has three different states (susceptible, exposed, and infected) (Figure 2). We refer to this structure as an SII $\times$ SEI model. These particular disease states are selected to reflect the HIV-TB co-epidemics, and we label the disease states accordingly. The first disease characterizes HIV, where the states are susceptible (S), infected with HIV but asymptomatic (I), and AIDS (A). We assume that the disease stage AIDS



Notes. The boxes represent cohorts of individuals and the arrows represent disease transmission, disease progression, maturation, or death. X_{SS} = susceptible to both diseases, X_{IS} = infected with HIV, X_{AS} = infected with AIDS, X_{SL} = infected with latent TB, X_{IL} = infected with HIV and latent TB, X_{AL} = infected with AIDS and latent TB, X_{ST} = infected with active TB, X_{IT} = infected with HIV and active TB, X_{AT} = infected with AIDS and active TB.

Figure 2: Schematic Diagram of the SII SEI Model

occurs at the onset of symptoms. The second disease represents TB, where the states are susceptible (S), latent tuberculosis (L), and active tuberculosis (T). In this model, $\beta.$ indicates the HIV sufficient contact rate, $\nu.$ is the HIV progression rate (varying according to TB status) and $\delta.$ is the TB progression rate (varying according to HIV status). Long et al. computes the basic reproduction ratio R_0 (the average number of secondary infection caused by a newly infected individual) and determine the conditions under which the disease free equilibrium (DFE) is locally and globally stable. The paper determines the quasi-disease-free equilibrium (QDFE), where infected individuals all have either HIV or TB, and illustrates, by numerical experiments, the conditions under which the co-infection equilibrium (CIE) exists. All of these results can be found in Long et al. (2008). The model can also be extended to co-epidemics with treatment where the treatment is identified by a reduction in AIDS-related death rate μ_A, transmission probability θ_A, consequently sufficient contact β_A, and disease progression rates $(\delta_{SL}, \delta_{IL}, \delta_{AL})$ and ν_{IL}.

Due to non-linear complexity, it is impossible to identify the population in each compartment at a given time analytically. Therefore, the paper presents an illustrative numerical analysis of HIV-TB co-epidemics in India by calculating the population in each compartment after 20 years. The analyses suggest that exclusive treatment of only one disease may substantially reduce that epidemic by decreasing disease prevalence and preventing new infections or deaths, but may exacerbate the other epidemic. This paradoxical result occurs because people co-infected with the second disease may live longer due to treatment, and subsequently may infect more people with the second disease.

Homogeneity in population and its behavior are naturally considered as the prerequisite assumption of an epidemic model of contact-transmission disease. However, Larson (2007) models influenza progression and its control within a heterogeneous population by difference equations. It shows that

population heterogeneity, mainly in the frequency and intensity of daily human-to-human contacts, and social distancing (people's intention to reduce the frequency and intensity of mutual contacts when influenza pandemic occurs) may affect disease progression significantly.

To discover the contributions made by a populations heterogeneity and social behavior in the evolution of the disease, Larson utilizes difference equations to model the day-to-day progression of the pandemic through the population. We assume that social contacts occur as a homogeneous Poisson process, with rate parameters dependent on the level of social activity (high and low). In particular, we define λ_H (λ_L) as contact rate of highly socially active (lowly socially active) people, $n_H(0)$ ($n_L(0)$) as the initial population of high-activity (low-activity) people (so $n_H(0)+n_L(0)$ is the total population which remains constant), $n_H^I(t)$ ($n_L^I(t)$) as population of high-activity (low-activity) infectious people at day t and $n_H^S(t)$ ($n_L^S(t)$) as population of high-activity (low-activity) susceptible people at day t. So we have that

$$n_j^S(t) = n_j^S(t-1) - n_j^I(t).$$

Define $\beta(t)$ as the probability that on day t a random interaction is with an infectious person, and we have

$$\beta(t) = \frac{\lambda_H n_H^I(t-1) + \lambda_L n_L^I(t-1)}{\lambda_H n_H^S(t-1) + \lambda_L n_L^S(t-1)}.$$

Let p be the probability that a susceptible person become infected, given contact with an infectious individual. It is derived in Lang's paper that the probability that a susceptible person in activity level j becomes infected on day t $p_j^S(t)$ has

$$p_j^S(t) = 1 - \mathrm{e}^{\lambda_j \beta(t) p}$$

and $p_j^S(t) \sim \lambda_j \beta(t) p$ if $\lambda_j \beta(t) p << 1$. This indicates that social activity frequency (λ_j) is just as important as a vaccine (p) in spreading or reducing the spread of the infection.

Larson also gets the basic reproductive ratio $R_0(t)$ at day t and verifies that the contribution of high-activity and low-activity groups to $R_0(t)$ are proportional to their contact rate. Larson (2007) also demonstrates, by numerical examples, that $R_0(t)$ is highly sensitive to t, which peaked at some-time and plummeted down to a level below 1 afterwards due to the high-activity susceptible population members becoming infected the being infectious and then leaving the circulating population.

To understand the role of reducing daily contacts, social distancing and hygienic factors in controlling influenza pandemic, the paper varies the model by letting $\lambda_i(t)$ dependent of the people in the circulation and adding $0 \leq d_j(t) \leq 1$ as the social distancing and hygienic factor which is applied to multiply $\lambda_j(t)p_j$, the primary determining factor associated with disease spread. Numerical experiments have also shown the importance of implementing social distancing and related hygienic controls early during the outbreak and keeping those measures in force for an extended period of time. Early implementation can reduce the eventual proportion of the infected population in each social-activity group significantly.

Bio-terror

Here we briefly deviate to describe a paper analyzing atmospheric release disease: anthrax. The paper also investigated a less stepped field: the emergency response of a government to bio-terror attack. In, Craft et al. (2005), the authors aimed to aid in understanding how best respond to a bio-terror anthrax attack by analyzing a system of differential equations of atmospheric release model, a spacial array of bio-sensors a dose response model, a disease progression model and a set of spatially distributed tandem queues for distributing antibiotics and providing hospital care. The paper is the

first to derive mathematical expressions for number of deaths resulting from an aerosol bio-terror attack with a noncontagious agent. Interestingly, the paper derives the fraction of death to infection analytically and illustrates that the fraction of infected people in the service zone who die is constant and is linear in the length of antibiotics distribution time and the efficacy of the antibiotics. The authors have also verified that the prophetic efficacy, of bio-sensors and and early symptomatics, is three times more effective than that of treatment.

Post-traumatic stress disorder

Post-traumatic stress disorder (PTSD) is a common disease among U.S. troops in Operation Iraqi Freedom (OIF) during each month of deployment. Atkinson et al. (2009) develops a dynamic model for this disease. Let $j = 1, 2, 3$ denotes active Army, reserve *Army* and Marines, respectively. The kth service member of type j has an indicator process $C_{kj}(t)$, $t = 1, 2, \ldots$ that characterizes his deployment history: $C_{kj}(t) = 1$ if service member k was deployed in month t and 0 otherwise. We will then show how service member k of type j develops PTSD as a function of $\{C_{kj}(t)\}_{t\geq 1}$.

Let $D_{kj}(t)$ be the random cumulative stress of member k of type j at the end of month t. The initial stress before the first month of deployment is an independent and identically distributed (i.i.d) exponential random variable with mean α^{-1}. Let $\{E_{kj}(t)\}_{t\geq 1}$ be independent random variables that represent the random stress that service member k of type j incurs during month t, if deployed that month. $E_{kj}(t)$ is assumed to be a compound Poisson variable with mean $\lambda_j(t)$ and batch size b.

We assume there is a geometric decay at monthly rate $\theta \in [0, 1]$ during the months when $C_{kj}(t) = 0$. $\tau_{kj}(t)$ is the month during which the deployment started if $C_{kj}(t) = 1$, and is the month during which the current break started if $C_{kj}(t) = 0$. Therefore, the stress dynamics are given as follows:

$$\begin{aligned} D_{kj}(t) &= D_{kj}(\tau_{kj}(t) - 1) + \sum_{s=\tau_{kj}(t)}^{t} E_{kj}(s), \text{ if } C_{kj}(t) = 1, \\ D_{kj}(t) &= D_{kj}(\tau_{kj}(t) - 1)\theta^{t-\tau_{kj}(t)-1}, \text{ if } C_{kj}(t) = 0. \end{aligned} \tag{2.1}$$

Denote $\bar{D}_{kj}$ as the threshold of service member k of type j, i.e. he gets PTSD if $\max_t D_{kj}(t) \geq \bar{D}_{kj}$. We assume that the distribution of $\bar{D}_{kj}$ is exponential with mean γ^{-1}. So, the probability that a service member gets PTSD with cumulative stress D is:

$$1 - e^{-\gamma D}.$$

Let m be the number of months deployed and t_m be the final month that the service member deploys. So the probability of developing PTST is:

$$\begin{aligned} &P(\max_t D_{kj}(t) > \bar{D}_{kj}) \\ &= P(D_{kj}(t_m) > \bar{D}_{kj}) \\ &= E[P(D_{kj}(t_m) > \bar{D}_{kj} | D_{kj}(t_m))] \\ &= E[1 - e^{-\gamma D_{kj}(t_m)}] \\ &= 1 - \exp(-\sum_{C_{kj}(t)=1} \frac{\lambda_j(t)}{b}(1 - e^{-\gamma b})). \end{aligned}$$

A service member with PTSD experiences a log-normal time lag to first develop symptoms after his cumulative stress level exceeds $\bar{D}$. This time highly depends on whether he is physically in the

military or has returned to civilian life. The random variables T_1 and T_2 represent a military and a civilian time lag, respectively. For $i = 1, 2$, T_i has pdf $f_i(t)$ and cdf $F_i(t)$, with mean e^{μ_i} and dispersion factor e^{s_i}. Let $S_j(t)$ denote the cumulative number of service members of type j who have developed PTSD symptoms by t and X_{kj} be the amount of time service member k of type j develops PTSD and the onset of symptoms. Hence, we have:

$$S_j(t) = \sum_k 1_{\{\bar{t}_{kj} + X_{kj} \leq t\}},$$

where $\bar{t}_{kj} = \min\{t | D_{kj}(t) \geq \bar{D}_{kj}\}$. The paper simulates the PTSD process with initial state and parameters obtained from real available data. It calculates the number of PTSD service members with and without symptoms as well as their proportions in each of the three types. For details, please refer to the original paper.

Coordination and competition

Coordination and competition in response to epidemics usually takes place in the distribution of vaccines in order to control epidemics. Like coordination issues in other cases, problems related to epidemics are also involved with the gap between local and global optimization. In other words, how should the participants in a game cooperate efficiently so as to align their individual interest with that of the entire system. We review 2 papers here one of which discusses supply chain coordination in influenza vaccination and the other deals with selfish drug allocation of countries in an international influenza pandemic.

Chick et al.(2008) makes a first attempt to relate a supply chain management model to an epidemic dynamics. It tries to present a model of a governments decision of purchase quantities of vaccines, which balances the public health benefits of vaccination and the cost of procuring and administering those vaccines, and a manufacturers choice of production volume. The optimal decisions of each can be characterized in both selfish and system-oriented play, and, thus, help us assess whether several contracts can align their incentives. Unlike usual supply chain models, this paper considers a supply chain where the uncertainty occurs at the production level, thus, converting the usual buy-back contracts into pay-back ones and revenue-sharing contracts into cost-sharing ones. You may refer to Yano et al. (1995) for a review of papers investigating yield uncertainty.

The government needs to decide the proportion of the population f to be vaccinated before the influenza occurs in autumn, and this decision will result in $T(f)$, determined by influenza dynamics, infected individuals by the end of the influenza season. Knowing the number of doses of vaccine Nfd, where N is the entire population and d is the number of doses a vaccinated person needs, the government orders, the vaccine manufacturer, in the face of a newsvendor problem, needs to decide the number of eggs n_E, each costing c, to produce. Each egg may produce U doses of vaccine, which is a random variable of mean μ and variance σ^2. The manufacturer's problem is characterized as follows:

$$\min_{n_E \geq 0} MF = E[cn_E - p_r Z],$$

where $Z = \min\{n_E U, fNd\}$, p_r is the price per dose of vaccine. And the government's problem can be identified as follows:

$$\min_{0 \leq f \leq 1} GF = E[bT(\frac{W}{Nd}) + p_a W + p_r Z],$$

where $W = \min\{Z, \bar{f}Nd\}$ such that

$$\int_0^{fNd/n_E} u g_U(u) du = \frac{c}{p_r}.$$

Here p_a is the administering cost per dose and $\bar{f} = \sup\{f : bT'(f) + p_a Nd < 0\}$, indicating the maximum fraction that the administering more vaccine is profitable and the integration condition is solution to the newsvendor problem faced by the manufacturer. The system-wide problem can be formulated as following:

$$\min_{0\leq f\leq 1, n_E\geq 0} SF = E[bT(\frac{W}{Nd}) + p_a W + cn_E].$$

The uncertainty in U results in vaccine manufacturer's risk to produce sufficient eggs and, thus, this Stackelberg game pushes the manufacturer to produce less than the systematic-optimized quantity. The paper discusses different coordinating contracts, like wholesale price and payback contracts, and verifies that only the whole-unit discount/cost-sharing contract, which keeps the relationship between the optimal production level and order quantity level linear, so as to encourage the government to vaccinate a higher fraction of population and the manufacturer to produce enough.

Most countries now store antiviral drugs in order to slow down the spread of influenza epidemic. Therefore, Sun et al. (2009) tries to study how each country make decisions to allocate its own stockpile in order to protect its population. It develops a two-period multivariate Reed-Frost model to represent the spread of the epidemic within and across countries at its onset. The model captures three critical sources of uncertainty: the number of initial infections, the spread of the disease, and drug efficacy. The paper analyzes the epidemic control issue where antiviral drugs or vaccines are distributed by strategic agents who seek to protect different parts of the population.

Consider $m+1$ countries $0, 1, 2, \cdots, m$ where country i's population size is N_i and the transmission probability from a non-treated infective in country i to a non-treated susceptible in country j is denoted by $a_{ij} \geq 0$. Access to drugs is assumed to decrease susceptibility by a factor δ, and infectiousness if infected by a factor ξ. Let us now consider a multi-dimensional Reed-Frost epidemic model of two periods. Let X_i^t represent the number of susceptibles in location i at time t who have not been treated, whereas $\bar{X}_i^t$ represents the number of susceptibles at time t who have taken the drug previously. Similarly, Y_i^t and $\bar{Y}_i^t$ represent the number of infectives in location i at time t who have (Y_i^t) and have not $(\bar{Y}_i^t)$ used the drug, respectively. So, we have $X_i^t = X_i^{t-1} - Y_i^t$ and $\bar{X}_i^t = \bar{X}_i^{t-1} - \bar{Y}_i^t$. Let K_i be the drug stock-pile of country i which satisfies $K_i \leq N_i$ and $K = \sum_{j=0}^m K_j \leq N_0$. Denote n_j^i as the number of country j's susceptibles treated by drugs from country i and $\mathbf{n}$ is the decision matrix $\{n_j^i\}_{(m+1)\times(m+1)}$ such that row vector $\mathbf{n}^i$ represents country i's decisions. Then country i seeks to maximize the average number of susceptibles, thus to minimize the average number of infectives, in its population corresponds to the following optimization problem:

$$\max_{\mathbf{n}^i} f(\mathbf{n}) = E_{Y_0^0}[E[X_i^2 + \bar{X}_i^2 | \{X_j^0, \bar{X}_j^0\} j \geq 0, Y_0^0]],$$

where $\sum_j n_j^i = K_i$ and $0 \leq n_j^i \leq N_j - \sum_{k\neq i} n_j^k$. We can show that for small-enough a_{kl} $k \neq l$, country i never gives drugs to country $j \neq 0$, i.e. country i only contributes drugs to country 0 only when between country transmission rates are sufficiently small. Additionally, when a_{kl} $(k \neq l)$ are sufficiently small, the optimal decision for country i is either to give up everything to country 0 or give nothing, depending on whether a_{ii} reaches a common threshold or not. In this case, the game is super-modular and a unique Nash equilibrium exists that is Pareto optimal. In the central planner case, the central planner allocates the world-wide drug inventory K to different countries. Suppose n_i is the amount of drugs allocated to country i and define $\mathbf{n} = (n_0, n_1, \cdots, n_m)$. The central planner's optimization problem is, thus,

$$\max_{K=\sum_j n_j} f(\mathbf{n}) = E[\sum_i^m (X_i^2 + \bar{X}_i^2) | \{N_j - n_j, n_j\}_{j\geq 0}].$$

Sun et al. (2009) have showed that when between-country transmission rates are low, the central planner always supply country 0 as much as possible. They have also verified that when N_i are big enough small, all countries would benefit from letting the central planner decide. We may alternatively considers the probability of no infection and is still able to reach similar results with the condition a_{kl} small enough replaced by $a_{0k}(k > 0)$ small enough.

To conclude, the papers that aim to understand the dynamics of epidemics usually utilize differential or difference equation systems and made some effort to capture the population of patient/healthy people at a given time. Due to non-linear complexity, it is generally impossible to expect an analytical expression. So, the researchers usually get the result through numerical experiment. The treatment or policy is usually realized by changing certain transmission or infection rate parameters. For the coordination problem in epidemics, papers usually combine an epidemic dynamics model with a game theory model. They use the epidemic model to capture the population of patients and the game theory model to quantify the participants' decisions and, eventually, to provide some insights on the connection between local and global optimization.

2.2 Vaccination Optimization

Vaccination against infectious disease is hailed as one of the greatest achievements in public health. Proper vaccination is a powerful weapon against a huge proportion of diseases. However, there are a few problems caused by the uncertainty in vaccine yield and the increasing complexity and intractability due to the expansion of diseases covered. The purpose of this subsection is to take a close look at the papers dealing with these 2 problems.

Vaccine yield uncertainty

Vaccine yield uncertainty usually arises from the 2 facts. First, the vaccine production process usually takes place in embryonated eggs, and the number of eggs needed must be anticipated well in advance. The actual yield $n_E U$ suffers from an inherent uncertainty regarding the quantity of vaccine obtained per chicken egg (U) since the uncertain growth of viral strains. Second, the antigenic drift of some viruses, such as influenza, requires the vaccines be reformulated every year. Wu el al. (2005) analyzed this phenomena and proposed that vaccine efficacy can be enhanced by taking into account the antigenic histories of vaccines. The annual vaccine-strains selection problem can be formulated as a stochastic dynamic program using the theory of shape space, which maps each vaccine and epidemic strain into a point in multidimensional space. Computational results show that a near-optimal policy can be derived by approximating the entire antigenic history by a single reduced historical strain, and then solving the multi-period problem myopically, as a series of single-period problems.

As as been discussed above, Chick et al. (2008) considers the supply chain coordination problem in the face of influenza vaccine yield uncertainty and shows that a wholesale-unit discount/cost-sharing contract can successfully align the incentives from the government and the vaccine manufacturer, thus coordinating the supply chain. The key idea underlying this optimal contract is that it successfully balances the manufacturer's risk of both excessive and insufficient yield. That is to say, the side effect of uncertainty in vaccine yield can be neutralized to the largest extent by this contract.

Deo et al. (2009) and Cho (2010) thoroughly investigates the impact of vaccine yield uncertainty to the influenza vaccine market and government's optimal choice of vaccine composition and deference. In Deo et al. (2009), the mismatch between demand and supply in the US influenza vaccine market is carefully investigated with the tool in economics: a two-stage game in oligopolistic competition. The paper particularly aims to understand the interaction between yield uncertainty and firms'

strategic behavior in production. A variation of Counot competition model enables us to see that yield uncertainty contributes to a high degree of concentration in vaccine industry and a reduction in vaccine output as well as the expected consumer surplus in equilibrium.

Suppose the vaccine industry consists of n firms, $\{1, 2, \cdots, n\}$. $\bar{q}_i$ is the quantity of eggs, with cost c_1 per each, produced by firm i, while its vaccine yield q_i equals $\alpha_i \bar{q}_i$, with cost c_2 per each, where $\{\alpha_i\}$ are i.i.d. random variables with expectation μ and variance σ^2. During the analysis, we keep μ constant. Therefore, we have the total production $Q = \sum_i \alpha_i \bar{q}_i$ and price $p = a - bQ$. The problem can be interpreted as a two-stage game, the first stage of which deals with the equilibrium number of firms in the market where there occurs a fixed cost of entry, while the second stage is to pursue the equilibrium target production quantities and profits given the number of firms. The solution of this game relies on backward induction.

From the notation and explanation above, we obtain the expected profit of the i'th firm is:

$$\Pi_i(\bar{q}_i) = E[(a - b(\sum_j \alpha_j \bar{q}_j))\alpha_i \bar{q}_i - (c_1 + \alpha_i c_2)\bar{q}_i].$$

To get the equilibrium condition $\bar{q}_i^*$, we simply need to take derivative with respect to $\bar{q}_i$ and equate it with 0. We can prove, for fixed n, that (1) both the target quantity $\bar{q}_i^*$ and the expected production quantity q_i^* are decreasing in $\delta = \sigma/\mu$, and (2) each firm's expected profit is first increasing and then decreasing in δ if $n > 3$, and monotone decreasing in δ if $n \leq 3$. In short, the yield uncertainty reduces the target egg quantity and total expected vaccine quantity brought to the market, while its impact on expected profit is a result of the trade off between this reduction and the resulted market price increase.

On the first stage of the game, we assume there exists an f as the entry cost for a firm. Suppose n^* is the equilibrium number of firms in the industry, we must have $\Pi_i^*(n^*) \geq f$ and $\Pi_i^*(n^* + 1) \leq f$, since otherwise entering firms are losing money or earning sufficient profits to attract additional entrants. Solving this 2 equations and we get that there exists an attractiveness factor $(a - c)/\sqrt{bf}$ where $c = c_1/\mu + c_2$, indicating the attractiveness of the market. When the market is not attractive enough ($(a - c)/\sqrt{bf}$ is not big enough), uncertainty decreases the number of firms at equilibrium. However, if the market is attractive enough, the small amount of uncertainty can result in entry of more firms. Based on the calculation of n^*, together with that of Π_i^*, we are enabled to show $E[Q_u^*] \leq Q_d^*$; i.e. the expected quantity produced by the market under yield uncertainty is no more than that in the deterministic case. We may also consider the decision problem of central planer who wants to maximize the total social welfare:

$$\max_{\bar{q},n} E[W(Q, n)] = \max_{\bar{q},n} E[\int_0^Q (a - bq)dq - cQ - nf], \tag{2.2}$$

where Q, and f are defined previously. For a complete analysis of this optimization and its comparison with the Cournot game case, please refer to the original paper. In conclusion, the analysis presented in Deo et al. (2009) shows that the interaction between yield uncertainty and market attractiveness.

Following the settings in Deo et al. (2009), Cho (2010) attempts to analyze the trade off the government is facing when deciding the composition of seasonal influenza vaccine. Cho mainly considers two key trade-offs. First, if the government decides to retain the current vaccine composition instead of updating to a new one, there is lower uncertainty in production yields, but the current vaccine could be less effective if a new virus strain spreads. Second, if the government decides early with less information, then manufacturers have more production time, but the reduced information increases the risk of choosing a wrong strain. To, simplify, we assume there are only two types of

virus strains. We can derive an optimal dynamic threshold policy for this decision. To explicate, the government 's decision on the following three options: (1) select the current vaccine strain s_1; (2) select the most prevalent new strain s_2; or (3) defer selection to the next period depends on the level of the prevalence estimate of strain 1 at each period. Because of the greater uncertainty in production yields of new vaccines, the optimal thresholds are neither symmetric between retaining and updating the composition nor monotonic over time. The analysis shows that the dynamic optimal policy can significantly improve the entire social welfare.

We consider the finite-horizon discrete-time model with period $t \in \{1, 2, \cdots, T\}$. We assume the overall efficacy of vaccine j, by $e_j(\theta) = e_{j1}\theta + e_{j2}(1-\theta)$, where $\theta \in [0,1]$ is the prevalence of s_1: the proportion of s_1 cases among all cases caused by either strain during the upcoming flu season, and e_{jk} denotes the efficacy of vaccine containing strain j against virus k. The true value of θ remains unknown until the end of the flu season, but the government develops the initial distribution of $\bar{\theta}_1$ based on previous information and update $\bar{\theta}_t$ with the help of new information it observes at period t. Let $\hat{\theta}$ be the mean of $\bar{\theta}_t$ in period t and $\Theta_{t+1}(\hat{\theta})$ be the pre-posterior mean in period $t+1$, and it is assumed that $E[\Theta_{t+1}(\hat{\theta})] = \hat{\theta}$. For vaccine yield uncertainty, we still apply the model in Deo et al. (2009) and let $\bar{q}_{t,j}$ be the quantity of eggs of strain j produced at t and y_j (of mean μ and variance σ_j^2) as the ratio of the number of vaccine doses to the number of eggs. $\sigma_1 < \sigma_2$ since the yield is more predictable when repeating the production of the vaccines the same as the last season. We assume the production capacity r be the maximum number of eggs the industry can produce in each period, so $r(T-t)$ is the total potential number of eggs at t: the early the decision on which to produce, the greater the amount of doses the industry can produce. We still assume the pricing function $p = a - bQ$ and let the optimal target quantity of vaccine doses of strain j be q_j^*. So, the expected doses supplied at t, $q_{t,j}$, is determined as $q_{t,j} = min\{r\mu(T-t), q_j^*\}$.

To identify the government's objective function, we denote $W_{t,j}(\hat{\theta})$ as the expected social welfare if the government selects strain j at time t when current estimate of θ is $\hat{\theta}$, then:

$$W_{t,j}(\hat{\theta}) = f(q_{t,j}, \delta_j)e_j(\hat{\theta}) - g(q_{t,j}, \delta_j),$$

where $\delta_j = \sigma_j/\mu$, f represents the social benefits of immunization and g denotes the social cost of vaccines, as is given, for example, by (2.2). Now, we are able to present a dynamic programing formulation of the government's decision at t, with estimate $\hat{\theta}$, $V_t(\hat{\theta})$. Define $W_t(\hat{\theta}) = \max\{W_{t,1}(\hat{\theta}), W_{t,2}(\hat{\theta})\}$. Then, for $t = 1, 2, \cdots, T-1$:

$$V_t(\hat{\theta}) = \max\{W_t(\hat{\theta}), EV_{t+1}(\Theta_{t+1}(\hat{\theta}))\},$$

with terminal condition $V_T(\hat{\theta}) = 0$. We are able to show that there exists $\theta_t^* \leq \theta_t^{**}$ for each t such that if $\theta_t^* < \hat{\theta} < \theta_t^{**}$, it is optimal to defer; if $\hat{\theta} \geq \theta_t^{**}$, it is optimal to select s_1; and otherwise, it is optimal to select s_2. Numerical example indicates that when t is big enough, we usually have $\theta_t^* = \theta_t^{**}$. To summarize, the analysis based on vaccine yield uncertainty has shown that there is an optimal threshold policy for when to retain the current strain, change to new strain, or defer. The greater yield uncertainty of a new strain leads to a smaller quantity of vaccines in equilibrium.

Vaccine selection

There is still a line of research that analyzes the vaccine selection and tractability problems. Papers contributing to this line usually analyze how to optimally select a combination of vaccines so as to minimize the full-vaccination cost and extra-immunization as well as how to ensure a child to receive timely coverage against vaccine-preventable diseases. Here we present a few papers devoted to this two aspects.

Hall et al. (2008) makes some attempt to analyze the pediatric vaccine formulary selection problem caused by the increasing complexity due to the expansion of the diseases covered. The objective of this paper is to minimize weighted sum of the cost (economic issue) to fully immunize a child and to minimize extra-immunization (medical issue) in pediatric immunization. Such problem is called the General Vaccine Formulary Selection Problem (GVFSP).

Let $T = \{1, 2, \cdots, \tau\}$ be the set of time periods for a given childhood immunization schedule, $D = \{1, 2, \cdots, \delta\}$ be the set of diseases, $V = \{1, 2, \cdots, v\}$ be the set of vaccines available. Also define K_v as the economic (c_v) or extra-immunization cost λ_d of vaccine $v \in V$, $I_v d = 1$ if v immunizes against disease d, 0 otherwise and $X_t v = 1$ if v is administered in time period t, 0 otherwise. So, $K_v = c_v + \sum_d \gamma_d I_{vd}$. We let m_{dt} be the minimum number of doses required for disease d in period t, M_{dt} be the maximum number and U_{dt} be the number of doses that have been administered for disease d at through period t. So, the optimization problem should be formulated as follows:

$$\min \sum_t \sum_v K_v X_{tv},$$

subject to $U_{dt} \leq U_{d(t-1)} + 1$, $U_{dt} \leq U_{d(t-1)} + \sum_v X_{tv} I_{vd}$ $m_{dt} \leq U_{dt} \leq M_{dt}$, $X_{tv} = 0, 1$ and $U_{dt} \in \mathbb{Z}^+$. Hall et al. (2008) has proved that this GVFSP is NP-hard and developed a dynamic programing (DP) algorithm. The NP-hardness of GVFSP implies the need of heuristics that are sub-optimal but executable within polynomial time. The paper presents us a heuristic of time complexity $O(\delta \tau v)$. It has been discussed in the paper that the DP approach might be used when there is need for exactly choosing a vaccine formulary schedule, while the heuristic approach is helpful if we want to efficiently analyze larger future childhood immunization schedules.

Engineer et al. (2009) aims to develop a decision support tool to help providers and caretakers in constructing catch-up schedules for childhood immunization. The authors enable the decision support tool to remove from the task of constructing catch-up schedules the tedious combinatorial aspects, while maintaining a level of generality that allows easy accommodation for changes in the existing rules and adding new vaccines to the schedule lineup. Given the current age of a child and their vaccination history (i.e., the number and timing of doses of each vaccine already administered), the catch-up scheduling problem is one of constructing a schedule for the remaining doses so that each dose is scheduled within the minimum and maximum age for that vaccine and dose, and the time separation between (not necessarily successive) doses of the same vaccine does not violate a certain minimum gap. So, it can be viewed as a multi-processor scheduling problem where a job corresponding to a particular dose of some vaccine to ensure that the doses are scheduled in sequence. We can state the catch-up scheduling problem as follows: given a feasible schedule s, find its best extension s^* with respectable to: (1) number of completable vaccination series, (2) number of scheduled doses, and (3) the total delay from the recommended age of administering the scheduled doses, in stated order of priority. For a precise mathematical formulation of the problem, you may refer to the original paper. Engineer et al. also have proved that the catch-up scheduling problem is NP-complete and identified a DP algorithm for solving it. By observing and exploiting the fact that the required separation between doses of the same vaccine is nondecreasing in the age some previous dose is administered, the authors derive dominance criteria that are sufficiently tight in practice to solve practically sized problems very quickly. An online software has also been developed to implement the decision support tool based on their algorithm.

2.3 Organ Transplantation

Organ transplantation is now the only treatment for several diseases at the latest level, such as chronic renal/liver failure. However, there might not be enough organ donor to supply all patients in need of transplantation. Therefore, how medical organizations (such as United Network of Organ Sharing, UNOS) are continuously facing the problem of how to allocate the donated organs efficiently and equally. Additionally, patients' choice of acceptance and refusal is also worth considering, which inevitably makes the related issues more intertwined. Usually, operations researchers apply dynamic programing, Markov decision process or queueing theory to analyze the quality-adjusted life of patients as an indicator of efficiency, but the issue of equity is much more difficult to quantify, and we will present a few papers that innovatively model this and related issues analytically.

The operations research literature also includes several studies that address some aspects of the organ allocation problem. One of the first papers in this area is by Ruth et al. (1985), who present a simulation model for the waiting list in Michigan. Righter (1989) formulates the organ allocation problem as a stochastic assignment problem (Derman et al. 1972) and develops properties of the optimal policy. David and Yechiali (1985, 1990, 1995) and David (1995) study several sequential decision problems that are motivated by organ transplantation, from the perspectives of both a potential recipient (1985) and a centralized decision maker. In recent studies that combine analytical and empirical research, Ahn and Hornberger (1996) and Hornberger and Ahn (1997) develop kidney acceptance policies for potential recipients that explicitly incorporate patient preferences and demonstrate that some patients can afford to be selective when making transplantation decisions. Pritsker (1998) describes a large-scale simulation model for the liver allocation system that is used by UNOS to compare alternative liver allocation policies.

Kidney allocation

Zenios et al. (2000) is one of the first to consider the kidney allocation problem from both efficiency and equity perspectives. A dynamic resource allocation problem is provided with the tri-criteria objective of maximizing the quality-adjusted life expectancy of transplant candidates (clinical efficiency) and minimizing two measures of inequity: a linear function of the likelihood of transplantation of the various types of patients, and a quadratic function that quantifies the differences in mean waiting times across patient types. The dynamic status of patients is modeled by a set of linear differential equations, and an approximate analysis of the optimal control problem yields a dynamic index policy. A surprising result from their simulation study is that the policy currently used by UNOS is not appreciably more efficient (in terms of quality-adjusted life years per patient) than the FCFT(First-Come First-Transplanted) policy; hence, if one views QALY (Quality Adjusted Life Years) as the primary efficiency measure and adopts the relative inequity viewpoint, then FCFT is preferable to UNOS.

Su et al. (2004) develops an M/M/1 queue model, with homogeneous patients and exponential reneging, to examine the patient's choice on the high rate of organ refusals in the kidney transplant waiting systems. In addition, unlike the standard M/M/1 model, each service instance is associated with a variable reward that reflects the quality of the transplant organ, and patients have the option to refuse an organ (service) offer if they expect future offers to be better. Under an assumption of perfect and complete information, it is demonstrated that the queueing discipline is a potent instrument that can be used to maximize social welfare. In particular, first-come-first-serve (FCFS) amplifies patients desire to refuse offers of marginal quality, and generates excessive organ wastage. By contrast, last-come-first-serve (LCFS) contains the inefficiencies engendered by patient choice and

achieves optimal organ utilization.

Patients in this system behave as rational economic agents and determine whether to accept or decline each offer based on the offers quality. Similarly, the medical planner overseeing the system determines who should be offered each organ. Each patients objective is to maximize his or her own total expected discounted QALYs, and hence the patient solves an optimal stopping time problem: when to accept an organ offer. On the other hand, the medical planner wishes to maximize the sum of the rewards for all patients and has two policy levers at its disposal: organ rationing and patient prioritization. That is, the planner can influence system outcomes by limiting access to certain organs (rationing) and by dynamically prioritizing the candidates on the waiting list.

Let patients arrive in a time-homogeneous Poisson process with rate λ, cadaveric organs arrive, independently, with rate μ. A patient leaves the waiting list either when he receives and accepts an organ offer or when he dies after an exponentially distributed amount of time with mean $1/\gamma$. We normalize $\mu = 1$. Let a patient be on one of the three states: dialysis (waiting list), post-transplant and death. Patients on dialysis receive a continuous pay-off at rate h per unit time; patients who die receive an instantaneous payoff d Further, a patient also receives a pay-off from transplantation. There is also a random variable X, which takes values in $(\underline{x}, \bar{x})$ with probability density f, reflecting the post-transplant total expected QALYs for the patient receiving the organ.

To derive the socially optimal outcome $V(n)$ measured by the total QALYs of n people on the wait-list, we let $b(n)$ be an organ acceptance threshold so that only organs with quality exceeding $b(n)$ are offered to a transplant patient. The induction of $V(n)$ and $b(n)$ can be obtained trough dynamic programing methods and the optimal threshold $b^*(n) = V(n) - V(n-1)$. We can also consider each patient's acceptance thresholds $\{a_k(n) : n \geq 1, n \geq k \geq 1\}$ interpreted as: When the queueing length is n, the patient in position k will only accept organs with quality no less than $a_k(n)$. Clearly we must have,

$$a_1(n) \geq a_2(n) \geq \cdots \geq a_n(n) \geq b(n).$$

For each strategy profile a, we can derive the total discounted expected pay-off for every patient in position k of a queue with length n, $V_k^a(n)$ dynamically. Let $a_k^F(n)$ be the optimal thresholds in the FCFS queue, and we are able to show that they are independent of queueing length n, so we denote them as $\{a^F(k)\}$. We can still show that, with thresholds $a^F(k)$, the social welfare is maximized only if $\lambda = 0$. i.e. treatment rationing is an ineffective way to control patient behavior, and that the system is inefficient under the commonly used FCFS rule. However, we shall show in the next section that social efficiency can be achieved if the priority rule is LCFS. For more details, please refer to the original paper. However, the social efficiency of LCFS should be treated with caution because of strategic difficulties associated with its implementation: without any form of monitoring, any person in line has the motivation to balk and reenter the system at the top of the line. This, again, raises a very complicated ethical issue. The paper also investigates a generalization of the model with different priorities: assume a patient is granted absolute priority with probability $p \in [0, 1]$, thus, its effective arrival rate is adjusted to λp. $p = 0$ corresponds to the FCFS case while $p = 1$ corresponds to the FCLS case. It can be shown that the patients become less selective as the priority parameter (p) increases and, as a result, as the threat of a reduction in their priority following an organ refusal becomes more severe.

Living and cadaveric donation

Living donors are a significant and increasing source of livers for transplantation, mainly because of the insufficient supply of cadaveric organs. Alagoz et al. (2004) makes the first attempt to consider

the problem of optimally timing a living-donor liver transplant to maximize the patients total reward, such as quality-adjusted life expectancy. They aim to seek a policy describing those health states in which the living-donor liver transplantation should occur, and those where waiting is the optimal action. We can formulate a Markov decision process (MDP) model in which the state of the process is described by patient health and derive structural properties of the MDP model, including a set of intuitive conditions that ensure the existence of a control-limit optimal policy.

The model is formulated as a standard discrete-time infinite-horizon discounted MDP, whose state space is $S = \{1, 2, \cdots, H+1\}$ (the smaller the number the healthier the patient and $H+1$ means death), consisting health states and action space consists of two actions: wait and transplant. Let $r(h, T)$ and $r(h, W)$ be the reward of wait and transplant, respectively, at health state h and $P(h'|h)$ be the transition probability from state h to h' without transplantation. The objective function $V(h)$ is the discounted reward at health state h. Let $r(h, l)$ be the reward of transplanting a liver of quality l when the patient is at health state h. So, we have:

$$V(h) = \max\{r(h, l), r(h, W) + \lambda \sum_{h'} P(h'|h)V(h')\}.$$

Under mild assumptions of increasing failure rate and reward bounds, there exists an optimal control limit policy. i.e. there exists a state j, such that $a^*(1) = a^*(2) = \cdots = a^*(j-1) =$ "W" and $a^*(j) = a^*(j+1) = \cdots = a^*(H) =$ "T".

Alagoz et al. (2007a) considers the cadaveric liver issue and generalizes the previous model to include the quality of livers $S' = \{1, 2, \cdots, L, L+1\}$ (the smaller the number the better the liver quality and $L+1$ means there is no liver available). The transition probability matrix is of the form $P(h', l'|h)$, the reward function is $r(h, l)$ and the wait pay-off is $c(h)$. So, the objective function becomes:

$$V(h, l) = \max\{r(h, l), c(h) + \lambda \sum_{h', l'} P(h', l'|h)V(h', l')\}.$$

The paper also provides the sufficient condition for the liver-quality-based and patient-health-based optimal control-limit policy.

In Alagoz et al. (2007b), continues this line of research by combining the previous 2 issues discussed. This study aims to inform a ESLD (end-stage liver disease) of whether to accept the living-donor liver, the cadaveric liver or to wait. The difference between choosing a living-donor liver and a cadaveric liver is (1) the initial reward choosing the living-donor liver is bigger than choosing a cadaveric liver of any quality but there is a penalty (disutility) $\rho(h)$ associated with using living donor when the patient is in state h; and (2) there exists an arrival process cadaveric livers which become useless after the arrival period while the living-donor can wait until your decision is made. So the state space is still (h, l) where h means the health of the patient and l is the quality of the cadaveric liver. The paper also gives a sufficient condition for the cases when the optimal policy is an at-most-three-region: for a given liver of quality l, there exists a health state $j(l)$ so that the optimal policy is to wait if and only if the health state of the patient is no worse than $j(l)$ while there exists an at-most-two-region liver based policy: for a given health state h, there exists a liver quality $i(h)$ so that it is optimal to choose a cadaveric liver if and only if its quality is no worse than $i(h)$ and $a^*(h, i(h)+1) = a^*(h, i(h)+2) = \cdots = a^*(h, L+1) =$ "W" or $a^*(h, i(h)+1) = a^*(h, i(h)+2) = \cdots = a^*(h, L+1) =$ "T_{LD}", where "W" means to wait and "T_{LD}" means choose living-donor. We also suggest you to read the original paper for a detailed analysis.

Transplant waiting list

Unlike the previous works which study the optimal policy of allocating and selecting donated organs, Sandikci et al.(2008) considers the benefit of creating a more transparent waiting list of patients eligible for cadaveric liver transplantation. The paper analyzes these benefits by modeling patient's accept/decline decision as a MDP where the state is described by patient health, liver quality and the rank of the patient in the wait list. Analogous to the above reviewed papers, the monotonicity of the value function and the sufficient condition for limit-control type of optimal policy are derived. The price of privacy is measured by the number of expected days lost due to the lack of complete waiting list information. Extensive numerical experiments based on clinical data indicate that this price is typically on the order of 5% of the optimal solution value. An estimate for the true price of privacy would be obtained by comparing a system (in which every patient has partial rank information as in the current allocation system and behaves optimally with this information) to a benchmark system (in which every patient has full rank information and behaves optimally). The current model is unable to provide an exact value for the societal price of privacy because if the waiting list were to become transparent, the organ offer probabilities would change substantially as the allocation system moved to a new equilibrium, thus making precise parameter estimation using existing data impossible. Rather, due to difficulties in identifying an equilibrium in either of these systems, we focus on a special case where only one patient, who is provided the waiting list information, is considered. As a result, the quantities we provide can be viewed as estimates for the true values.

To summarize, organ transplantation issues are usually involved with the interplay between organ allocation, from the organization's perspective, and organ selection, from the patient's point of view. Ethical concerns like privacy and equity are also widely investigated in recent literature. Queueing and MDP methods are typical approaches to this type of problems.

There are also a few other papers that do not fall into one of the categories above. For example, Bersimas et al. (2008) develops an algorithm based on regression methods to predict the future health-care costs. Keeney (2008) discusses the relationships between personal decisions and premature deaths and suggests that more effort directed toward improving personal choices regarding life risks may be an effective and economical way to save lives.

3 Health-Care Institutions Operations

So far, our discussion have been confined to the macro-scale policy level issues in heal-care systems, i.e. the government/medical organization's optimal decision. We now turn to micro-scale management issues faced by medical institutions, like clinics, hospitals. Within our expectation, papers devoted to optimize the strategy of an individual institution contributes to a major part of the literature in health-care operations. Compared with policy level and individual level issues, the institution level problems link these two extremes and are, thus, more resourceful. Further, OR/OM literature usually analyzes the optimal decision of a firm, whose condition and behavior share some characteristics with a medical institution. Therefore, a great many of ideas, models and methods can be borrow from, with comparatively slight variation, the vast tradition OR/OM literature. In this section, we mainly discuss three groups of papers in this section: (1) capacity management, which deals with, for example, how to manage limited medical resources such as beds and operating rooms; (2) medical appointment scheduling, which seeks to improve medical service efficiency and quality under drastic demand uncertainty and no-shows; and (3) heal-care related manufacturing and supply chain

problems. A few other papers not included in these two groups, such as the revenue management for non-profit operations in hospitals, are reviewed at the end of this section.

3.1 Capacity Management

For most hospitals, capacity management is an inevitable problem. Crucial medical resources like diagnostic devices, ICU (incentive care units) beds and operating rooms are always in intensive need by patients in the wait list. Therefore, efficiently allocate/expand these resource is of the common interest shared by the medical institution and patients. For a review of this type of literature, please refer to Green (2004). In this subsection, we discuss how do fulfill this goal with the help of OR/OM models, beginning with a few papers discussing operating room allocation problem.

Operating room allocation

Operating room is at the frontier of resource limitation in hospitals. Cardoen et al. (2010) which briefly reviews operating room planning and scheduling problems provides a schematic picture of this area. But we only discuss a few recent OR papers that typically and efficiently models this and related issues. Lovejoy et al. (2002) analyzes how to expand operating room capacity efficiently. The paper investigates the trade-offs among three performance criteria: waiting time, scheduled procedure reliability, and hospital profits, which are of particular importance to three different constitutes of this problem: patients, surgeons and surgical staffs and hospital administrators. The objective is to determine how the hospital can best expand its capacity, acknowledging the key role that each constituency plays in that objective.

We assume that the hospital has m operating rooms, whose patients arrive via a Poisson process of rate λ. The operation procedure lengths are assumed to be i.i.d random variables with distributed as X. The decision variables are the number of cases to schedule per OR per day (n), which is a daily capacity decision, and the probability that a scheduled procedure begins on time (π). These choices, along with the parameters of the regular-time day length (T), the average margin per case (R) generated excluding operating costs, and the average costs of regular time and overtime staffing (C_r and C_{ot}, respectively) in the operating rooms will determine the daily profits generated in the operating rooms (details below). We assume that T will be chosen optimally given n and. Also, for a given arrival rate the caseload per day, n, will uniquely determine the capacity of the system and hence the wait (W) to get on the schedule. Therefore, a specific choice of n and π will determine the three performance metrics of interest to our stakeholders (π (both as a decision variable and a performance metric), W and total profit). Suppose that there are k procedures scheduled in a specific room. Let t_i denote the scheduled start time and S_i denote the (random) actual start time for procedure i, $i = 1$ to k. Let X_i denote the random length of procedure i (by assumption, the random variables X_i have a common distribution), and let E_i denote the ending time for procedure i. The distribution of S_1 is assumed given and the same for all rooms. The choice of π, and the common distribution of procedure length (F_X), the statistics of all subsequent procedures (up through case k) are preordained. Specifically, $E_1 \sim S_1 + X_1$. Given the distribution of E_1 t_2 is set so that $F_{E_1}(t_2) =$ to ensure the desired start-time reliability for Case 2. Then, the second procedure starts either at its scheduled time or whenever the first procedure ends, that is $S_2 \sim E_1 \wedge t_2$. Working forward we have $E_i \sim S_i + X_i$ and $S_{i+1} \sim E_i \wedge t_{i+1}$for each i. To characterize the cost and revenue, the hospital needs to decide the regular hour T per day. Hence the expected daily cost is: $Cost(T) = C_r T + (C_r + C_{ot}) \sum_{i=1}^{n} p_i \int_T^{+\infty} (x - T) dF_{E_i}(x)$, where p_i is the stationary distribution of

i patients per-operating-room. And the profit of the total m operating rooms will be:

$$Profit(T) = R\lambda - mCost(T).$$

We can show that, given n (thus W) fixed, increasing π will generally decrease overtime costs, but these cannot decrease sufficiently to decrease total costs, and, thus, will reduce the total profit. If we fix π, instead, assigning more operating rooms will, intuitively, reduce W, but its impact on total profit is more complicated. For the detailed analysis and numerical approximations, please refer to the original paper.

Olivares et al. (2008) optimizes a hospital that balances the costs of reserving too much versus too little operating room capacity to cardiac surgery cases. The problem is of newsvendor type with heterogeneity in uncertainty and cost parameters. Their results reveal that the hospital places more emphasis on the tangible costs of having idle capacity than on the costs of schedule overrun and long working hours for the staff. The paper develops statistical methods that give consistent estimates of the model primitives, and derive their asymptotic distribution, which is useful to do hypothesis testing.

Deton et al. (2010) makes some effort to address the issue of optimal allocation of surgery blocks to operating rooms uncertainty. This is a challenging combinatorial optimization problem with additional complication in the uncertainty of surgical procedure duration. In this paper, we present stochastic optimization models for the assignment of surgeries to operating rooms on a given day of surgery. The objective includes a fixed cost of opening operating rooms and a variable cost of overtime relative to a fixed length-of-day.

There are two (1) how many ORs to open on a given day and (2) which OR to assign to each surgery in a daily listing. We assume that there are n blocks of surgeries $i = 1, 2, cdots, n$ and m operating rooms: $j = 1, 2, cdots, m$. $d_i(\omega)$ (as a random variable) is the duration of surgery block i, x_j is a binary decision variable indicating whether operation room j, y_{ij} is the binary decision variable representing overtime for operating room j and $o_j(\omega)$ is the overtime for operating room j. Let T be the time session considered in the problem, c^f be the fixed cost to open an operating room and c^v is the variable cost per unit time of overtime. Then the expected time can be formulated as follows:

$$E[(\sum_{i=1}^{n} y_{ij} d_i - T)^+].$$

As a result, the two-stage stochastic recourse problem can be formulated as following:

$$Z_S^* = \min\{\sum_{j=1}^{m} (c^f x_j + E[c^v o_j])\},$$

where $y_{ij} \leq x_j$, $\sum_{j=1}^{m} = 1$, $\sum_{i=1}^{n} d_i(\omega) y_{ij} - o_j(\omega) \leq T x_j$ a.s., $x_j, y_{i,j} \in \{0, 1\}$ and $o_j(\omega) \geq 0$. The decisions whether to open a given operation room (x_j) and which surgery block to allocate to each operating room (y_{ij}) are first-stage optimization decisions. The second-stage recourse decisions, overtime $o_j(\omega)$ for each operating room j are simple recourse decisions, which are easily solved given the random out come ($d_j(\omega)$) and the first-stage decisions x_i, y_{ij}. Both L-shape decomposition method in integer programing and a heuristic inspired by the newsvendor setting can solve this problem and we leave the details for the readers to refer to the original paper.

Like operating rooms, many intensive care units (ICU) face overcrowding. One response to such overcrowding is to bump ICU patients to other alternative departments to make room for new arrival. Dobson et al.(2008) develops a Markov chain model to help planners predict performance under

different arriving patterns. The state space S of this Markov chain consists of $n-$dimensional vectors $(s_1, s_2, \cdots, s_C)$ (the components are ordered in decreasing order, and $s_1 \leq D$), indicating the length of stay (LOS) of the patient on each bed in the ICU, where C represents the number of beds. On each day i, N_i ($N_i \leq M$) is the number of new arrival patients. If $N_i = n$, let L_j be the (i.i.d and independent of N_i) LOS of patient j with $L_j \leq D$. For an arrival pattern $\mathbf{a} = (a_1, a_2, \cdots, a_n) \in A$, we assume they are in decreasing order and $a_1 \leq D$, where A is the space of all vectors with dimension no greater than M. So, we have $p(\mathbf{a}) = c(\mathbf{a})P(N_i = n)\Pi_{j=1}^{n} P(L_j = a_j)$, where $c(\mathbf{a})$ is the number of unordered vectors with the same components as $\mathbf{a}$. We assume that if it is necessary to bump k patients, we remove the k patients with the least remaining days left in their stay (least time remaining bumped first). If two patients have the same number of days left and only one must leave, we pick arbitrarily. When bumping, we do not distinguish between newly arriving or existing patients; rather, we distinguish patients by their remaining LOS. We are assuming that the days remaining is a proxy for the health of the patient and that it is preferable to bump healthier patients rather than sicker ones. Note that we do not assume that the doctors remaining stay for each person currently in the ICU will making the bumping decisions know exactly the remaining LOS of each patient but rather they are capable of rank ordering the patients in terms of remaining LOS. We define p^O the probability an outlier ($s_j = D$) patient remains an outlier the next day. Define $o(\mathbf{s})$ as the number of outliers of state $\mathbf{s}$. The probability of u outliers of $o(\mathbf{s})$ staying as outliers can be computed as $P_o(u, o(\mathbf{s})) = \binom{n}{2}(p^O)^u(1-p^o)^{o(\mathbf{s})-u}$ if $u \leq o(\mathbf{s})$ and 0 otherwise. To help track the state transition dynamics, we define an operator $f_k(\mathbf{a}, (\mathbf{s}, u))$ that simulates the bumping process with state $\mathbf{s}$ and arrival pattern $\mathbf{a}$. Let $f_k(\mathbf{a}, (\mathbf{s}, u))$ be a $k-$vector obtained from concatenating the vector $\mathbf{a} = (a_1, a_2, \cdots, a_n)$ of length n and the vector $(s_1, s_2 \cdots, s_u, (s_{u+1} - 1)^+, c \ldots, (s_C - 1)^+)$ of length C into a single vector of length $n + C$, then sorting it in decreasing order and finally truncating it by only taking the first k components. Therefore, given $\mathbf{s}$, $\mathbf{a}$ and u, the next state of the system is $f_C(\mathbf{a}, (\mathbf{s}, u))$. We are now able to define the Markov chain's transition matrix $\mathbf{P}$. Let $\mathbf{s}, \mathbf{t} \in S$, and define $A_{(\mathbf{s},u)\mathbf{t}} = \{\mathbf{a} \in A : f_C(\mathbf{a}, (\mathbf{s}, u)) = \mathbf{t}\}$. The probability of making a transition from $\mathbf{s}$ to $\mathbf{t}$ is:

$$P_{\mathbf{st}} = \sum_{u=0}^{o(\mathbf{s})} P_o(u, o(\mathbf{s})) \sum_{\mathbf{a} \in A_{(\mathbf{s},u),\mathbf{t}}} p(\mathbf{a}).$$

We can use the aggregation-disaggregation to calculate the stationary distribution, π, of this Markov chain, which outperforms the traditional Gauss-Seidel iterative method in efficiency and as a deterministic algorithm. Numerical experiments, applying this method, to evaluate the performance, measured by bumping probability of arrival, number of remaining days for a bumped patient and the utilization of the ICU capacity, of an ICU suggest that smoothing the surgical schedule can have a more significant impact on bumping rates than increasing capacity.

Inpatient beds during demand surges

Many hospitals face the problem of insufficient capacity to meet demand for inpatient beds, especially during demand surges. This results in quality degradation of patient care due to large delays from admission time to the hospital until arrival at a floor. In addition, there is loss of revenue because of the inability to provide service to potential patients. A solution to the problem is to transfer patients between floors in anticipation of a demand surge. Thompson et al.(2009) poses an optimal reallocation problem that can be modeled as a finite-horizon Markov decision process. Due to the large number of patient categories and policy choices and the randomness of patient arrival and departure, the problem is very challenging. The system state at any point in time is represented, for each patient category, by the number waiting and the number being cared for on each floor.

We consider a decision process with a finite time horizon divided into m time periods of constant length and the transition probability is of period m, so the state process must also have time period index t. Let c_j be the maximum capacity of floor $j \in F$, $F_i \subset F$ be the set of feasible floors for patients of category i, a_{ij} be the reward from assigning a category i patient to floor j, and b_{ijk} be the transfer cost of category i patient from floor j to floor k. Let x_{ij} be the number of category i patients on floor j, while x_{i0} be the number of category i patients not on any floor. Let the arrival variable g_{it} be the number of category i patient arrivals during period t, the departure variable d_{ijt} be the number of category i patient departures from floor j. The decision variables are: $y_{i0j} \geq 0$, the number of category i unassigned patients to be assigned to floor j, and $y_{ijk} \geq 0$, the number of category i patients to be transferred from floor j to floor k. Let the minimum expected $n-$stage cost in state S be $V_n(S)$. So, we have that the decision variables must satisfy: (1) $\sum_j y_{i0j} \leq x_{i0}$ (waiting patients), (2) $\sum_i (x_{ik} + y_{i0k} + \sum_j y_{ijk} - \sum_j y_{ikj}) \leq c_k$ (floor capacity), (3) $y_{i0k} = y_{ijk} = 0$ if $k \in F \backslash F_i$. For a decision Y consists of $y'_{i0j}s$ and $y'_{ijk}s$, the stage associated cost is:

$$C(Y) = \sum_i \sum_k (-a_{ik} y_{i0k} + \sum_j b_{ijk} y_{ijk}).$$

Let $S = [X, t]$ be the current state and $\hat{S}$ be the state after arrivals and departures occur with decision Y. So the transition probability from S to $\hat{S}$, given decision Y is:

$$P_{S\hat{S}}(Y) = 1_{\{\hat{t}=(t+1) mod\ m\}} \pi P[g_{it} = \hat{x}_{i0} - x_{i0} + \sum_k y_{i0k}] \Pi_i \Pi_j P[d_{ijt} = -\hat{x}_{ij} + x_{ij} + y_{i0j} + \sum_k (y_{ikj} - y_{ijk})].$$

Now, we are able to give $V_n(S)$ as follows:

$$V_1(S) = \min_Y C(Y),$$

$$V_n(S) = \min_Y \{C(Y) + \sum_{\hat{S}} P_{S\hat{S}}(Y) V_{n-1}(\hat{S})\}.$$

To solve the above MDP problem, we may use the approximation methodology, for details, please refer to the original paper. Chao et al. (2003) also considers a patient switching problem by modeling it as a multi-site service systems with inter-site customer flows.

Diagnostic service management

Managing diagnostic services needs to strike a balance between accuracy of advice, callers' waiting time, and staffing costs by setting the appropriate capacity and service path. Hence, this is also a widely investigated problem by researchers in the filed of OR/OM. Green et al. (2006) considers the problem of managing patient demand for diagnostic service which consists of three parts: (1) outpatients, who are scheduled in advance; (2) inpatients, whose demands are generated randomly during the day; and (3) emergency patients, who must be served as soon as possible. The management of a diagnostic facility consists of two interrelated tasks: establishing an appointment schedule for outpatients and designing a system of dynamic priority rules for admitting patients into service in real time. The paper models the operations of a medical diagnostic facility with several patient types as a dynamic stochastic control problem and establishes structural properties of an optimal real-time capacity allocation policy under an arbitrary outpatient appointment schedule.

Consider N identical service periods in each working day, some of which may be reserved through appointment system. The schedule of accepted appoints is expressed as an $N-$dimensional binary vector a: $a_i = 1$ if the ith appointment slot has been filled and 0 otherwise. Due to the relatively low intensity of inpatient and emergency demands, no more than one request for each type of service

arrives during each period, and its arrival probabilities are denoted by p_i and p_e, respectively. There is a positive "no-show" probability for each out-patient schedule.

We are now at the stage to model the dynamics of the diagnostic facility as a Markov chain. The state of the system consists of the number of (non-scheduled) inpatients and (scheduled) outpatients, (n_i, s_i), after the action but waiting for service. It is assumed that the number of each type waiting during service slot $i+1$ can be higher than those waiting during slot i by at most 1.

Diagnostic facility collects r_s and r_n as a revenue for each outpatient and in patient, respectively. Delaying a service request incurs a waiting cost period w_s and w_n for out patients and inpatients, respectively. Finally, there is a penalty function $f(n,s)$ associated with patients not served by the end of the day and we can simply take π_s for each outpatient and π_n for each inpatient. To crack reality, assume: (1) $r_s > r_n$, (2) $w_s > w_n$, and (3) $\pi_s < \pi_n$.

With the notations and assumptions above, we can formulate the profit maximization problem. For a given schedule a, $V_i^a(n,s)$ is the optimal total expected profit over the $(N-i)-$period planning, from i to N, when the state after period i is (n,s). Then, we have:

$$\begin{aligned} V_i^a(n,s) = -sw_s - nw_n + p_e p_n[(1-p_s a_{i+1})V_{i+1}^a(n+1,s) + p_s a_{i+1} V_{i-1}^a(n+1,s+1)] \\ +p_e(1-p_n)[(1-p_s a_{i+1})V_{i+1}^a(n,s) + p_s a_{i+1} V_{i+1}^a(n,s+1)] \\ +p_n(1-p_e)[(1-p_s a_{i+1})H_{i+1}^a(n+1,s) + p_s a_{i+1} H_{i+1}^a(n+1,s+1)] \\ +(1-p_n)(1-p_e)[(1-p_s a_{i+1})H_{i+1}^a(n,s) + p_s a_{i+1} H_{i+1}^a(n,s+1)], \end{aligned}$$

where H_i^a is the maximization operator: $H_i^a = \max[V_i^a(n-1,s) + r_n, V_i^a(n,s-1) + r_s]$, if $n \geq 1$, $s \geq 1$; $V_i^a(n-1,0) + r_n$, if $n \geq 1$, $s = 0$; $V_i^a(0,s-1) + r_s$, if $n = 0$, $s \geq 1$; and $V_i^a(0,0)$ if $n = s = 0$. The boundary condition is given by $H_{N+1}^a(n,s) = V_{N+1}^a(n,s) = f(n,s)$. The diagnostic facility can maximize its profit by strategically choosing a. So, the outpatient appointment problem can be formulated as:

$$V^* = \max_a [V_1^a(0,0)].$$

Green et al. (2006) establishes that for any given service slot and specified number of patients of a given class (e.g., outpatients), it will be optimal to serve that class only if the number of the other class (e.g., inpatients) is below a critical value. However, this critical value increases as the number of outpatients increases. Therefore, the optimal policy assigns service priority so as to balance the congestion due to the two patient classes. A mild condition is provided in the paper to ensure a threshold type optimal policy, A linear approximation heuristic informs us that outpatients are served at the beginning of the day ($i \leq i_h^*$) and inpatients at the end of the day ($i > i_h^*$).

Patrick et al. (2008) presents a method to dynamically schedule patients with different priorities to a diagnostic facility. The objective is to achieve wait-time targets in a cost-effective manner. The problem is, again, modeled as a Markov decision process. Since the state space is too large for direct solution, we are only able to solve the equivalent linear program through approximate dynamic programing.

We consider a system that has the capacity to perform C_1 fixed-length procedures each day. At a specific point of time in a day, referred to as the decision epoch, the scheduler observes the number of booked procedures on each future day over an $N-$day booking horizon and the number of cases in each priority class to be scheduled. Demand arises from two sources, inpatients and outpatients. In practice, most inpatient demand is known at the beginning of each day once morning rounds have been completed on the wards. Outpatient demand arrives throughout the day, and thus is not completely known and prioritized until the end of the day. We assume that all decisions are made

once inpatient demand has been determined.

Assume $s = (\mathbf{x}, \mathbf{y}) = (x_1, x_2, \cdots, x_N; y_1, y_2, \cdots, y_I)$, where x_n is the number of patients already booked on day n, y_i is the number of priority i patients waiting to be booked. Let the set of all feasible s be S. The scheduler decides at each decision epoch which available appointment slots to assign to each unit of waiting demand and whether to divert patients to alternative capacity (surge capacity) at an additional cost. To capture the real case, we impose a limit C_2 to the number of patients diverted. A vector $(\mathbf{a}, \mathbf{z}) = \{a_{in}, z_i\}$, where a_{in} is the number of priority i patients to book on day n and z_i is the number of diverted patients of priority i. The feasible actions must insure that the base capacity is not exceeded, that no more than C_2 patients are diverted and that the number of bookings and diversions does not exceed the number waiting. Hence, they satisfy the following constraints: $x_n + \sum_{i=1}^{I} \leq C_1$, $\sum_{i=1}^{I} z_i \leq C_2$ and $\sum_{n=1}^{N} a_{in} + z_i \leq y_i$. We assume demand for each day is independent and each day's demand is independent. Thus, for a state s, we denote A_s as its feasible action set. Obviously, we have, if the number of new arrivals is represented by $\mathbf{y}'$ and the action is represented by $(\mathbf{a}, \mathbf{z})$, the probability of state transition from $(x_1, x_2, \cdots, x_N; y_1, y_2, \cdots, y_I)$ to $(x_2 + \sum_{i=1}^{I} a_{i2}, \cdots, x_N + \sum_{i=1}^{I} a_{iN}, 0; y_1' + y_1 \sum_{n=1}^{N} a_{in} - z_1, \cdots, y_I' + y_I - \sum_{n=1}^{N} a_{In} - z_I)$ is $p(\mathbf{y}') = \Pi_{i=1}^{I} p(y_i')$, where $p(y_i')$ is the probability that y_i' priority i patients arrive on a given day. A cost associated with booking a patient beyond the priority-specific wait-time target, a cost associated with using surge capacity, and a cost associated with demand that was neither booked nor diverted contribute to the total cost of an action. We write the costs as:

$$c(\mathbf{a}, \mathbf{z}) = \sum_{i,n} b(i,n) a_{i,n} + \sum_{i=1}^{I} d(i) z_i + \sum_{i=1}^{I} f(i)(y_i - \sum_{n=1}^{N} a_{in} - z_i),$$

where $b(i,n)$ is the cost of booking a priority i patient on day n, $d(i)$ is the penalty for diverting a priority i patient, and $f(i)$ is the cost associated with delaying a priority i patient's booking. For a explicit expression of $b(i,n)$, please refer to the original paper. Clearly, the cost function explicitly balance the cost to the patient in wait time and the cost to the system in having to resort to surge capacity.

For a state $(\mathbf{x}, \mathbf{y}) \in S$, we have the value function $v(\mathbf{x}, \mathbf{y})$ satisfies the following Bellman Equation:

$$\begin{aligned} v(\mathbf{x}, \mathbf{y}) = \min_{(\mathbf{a},\mathbf{z}) \in A_{(\mathbf{x},\mathbf{y})}} \{ c(\mathbf{a}, \mathbf{z}) + \gamma \sum_{\mathbf{y}' \in D} p(\mathbf{y}') v(x_2 + \sum_{i=1}^{I} a_{i2}, \cdots, x_N + \sum_{i=1}^{I} a_{iN}, 0; \\ y_1' + y_1 \sum_{n=1}^{N} a_{in} - z_1, \cdots, y_I' + y_I - \sum_{n=1}^{N} a_{In} - z_I) \}, \end{aligned}$$

where γ is the daily discount factor and D is the all possible incoming demand stream. As discussed above, the solution to this problem and the optimal policy can be obtained by linear value function. For more details, please refer to the original paper.

Modeling the design of diagnostic service centers as a multiple-server queueing system, with the servers performing a sequential testing process and the customers deciding whether or not to use the service, Wang et al. (2010) has found the dual concerns of accuracy and congestion lead to a counterintuitive impact of capacity: Increasing capacity might increase congestion. In addition, (1) patient population size is an important driver in management decisions, not only in staffing but also in accuracy of advice; (2) increasing asymmetry in error costs may not increase asymmetry in the corresponding error rates; and (3) the error costs for the two major stakeholdersthe service manager and the patientmay impact the optimal staffing level in different ways.

To simplify, we assume that the pathology θ of the patient is either "healthy", indicated by $\theta = -1$ or "sick", indicated by $\theta = +1$ and that $\theta = +1$ with probability π. π is assumed to be common knowledge and identical for all patients. There are two treatment options for each patient: "self-care" (A) or "visit the Emergency Department (ED)" (B). The appropriate treatment of pathology -1 is A and $+1$ is B, and the right treatment selection incurs no error costs. However, if a $+1$ (-1) patient wrongly selects A (B), he will incur a cost of c_A (c_B, respectively). Additionally, each nurse will cost c_n per unit of time. The error costs of the Health-Organization (HO) is C_A and C_B, analogously.

To characterize the diagnostic process, we assume that, for a patient pathology θ, the nurse observes, through questions and answers, a Brownian motion (BM) with drift θr and variance σ^2: $Y_\theta(t) = \theta r t + \sigma W_t$, where r implies the nurse skill level and W_t is the standard BM. For $y \leq 0 \leq x$, the nurse keeps asking questions until $Y_\theta(t)$ hits y or x. Let $\tau := \inf\{Y_\theta(t) \notin (y, x)\}$, and the patient is advised to seek treatment B (or A) if $Y_\theta(t) = y$ (or x). The stopping boundary vector $\mathbf{x} = (x, y)$ is referred to as the certainty threshold or the service depth set.

The enrolled members of the Health Organization (HO) fall ill according to a Poisson process of rate Λ and their aggregate rate is λ ($\leq \Lambda$). The performance of this service center is measured by both the probability of mis-diagnosis (error probabilities) and the patient waiting time. The error probabilities are denoted by α (= the probability that Y_{+1} hits y before x) and β (= the probability that Y_{+1} hits y before x). We refer to α (β) as the type I (II) error. To explicate, α is the probability of advising self-care to a sick patient (under-referral rate) and β is the probability of advising a healthy patient to go to the ED(under-referral rate). Clearly, α, β are functions of ($\mathbf{x}$). The service delivery process is modeled as an $M/G/m$ queueing systems, where m is the number of nurses. Each nurse performs the diagnostic process adopting the same threshold $\mathbf{x}$ with the same skill level r. We denote the expected waiting time as W.

Without the advising nurse, the patient's expected error cost is $\frac{1}{2}[(1-\pi)c_B + \pi c_A]$, since the probabilities of both type I and type II errors are 0.5. For a given threshold $\mathbf{x}$, the expected "post-call" cost is $(1-\pi)\beta(\mathbf{x})c_B + \pi\alpha(\mathbf{x})c_A$. So the patient's expected saving, with waiting costs excluded, is:

$$\Delta_P(\mathbf{x}) = (1-\pi)(\frac{1}{2} - \beta(\mathbf{x})c_B) + \pi(\frac{1}{2} - \alpha(\mathbf{x}))c_A.$$

The HO's saving is:

$$\Delta_{HO}(\mathbf{x}) = (1-\pi)(\frac{1}{2} - \beta(\mathbf{x})C_B) + \pi(\frac{1}{2} - \alpha(\mathbf{x}))C_A.$$

If we denote the expected waiting time in the queue is $W(\lambda; \mathbf{x}, m)$, so the utility of a patient calling the nurse line is:

$$U(\lambda; \mathbf{x}, m) = \Delta_P(\mathbf{x}) - c_w W(\lambda; \mathbf{x}, m),$$

where the waiting time cost rate is c_w. Each patient has a mixed strategy of calling probability p; i.e. the patient calls the nurse line with probability p. The homogeneity of patients imply that the equilibrium is of symmetric type. Hence, given $\mathbf{x}$ and m, all patients have the same calling probability $p_e(\mathbf{x}, m)$ and we have that $\lambda_e(\mathbf{x}, m) = p_e(\mathbf{x}, m)\Lambda$. Wang et al.(2010) shows, (1) if $U(\Lambda; \mathbf{x}, m) > 0$, $p_e(\mathbf{x}, m) = 1$; (2) if $U(0; \mathbf{x}, m) \geq 0 \geq U(\Lambda, \mathbf{x}, m)$, $p_e(\mathbf{x}, m) \in [0, 1]$; and (3) $U(0; \mathbf{x}, m) < 0$, $p_e(\mathbf{x}, m) = 0$. The HOs profit rate generated by the nurse line is defined as the difference between the rate of benefit (cost savings) from a nurse line and the staffing cost rate:

$$J(\mathbf{x}, m) = \Delta_{HO}(\mathbf{x}, m)\lambda_e(\mathbf{x}, m) - c_n m.$$

So the optimal decision $(\mathbf{x}^*, m^*)$ of the HO is defined to solve:

$$J^* = \max_{\mathbf{x}, m \geq 0} J(\mathbf{x}, m).$$

We can calculate the error probabilities α, β, the expected waiting time W and the equilibrium probability p_e by martingale and heavy traffic approximation methods. We can show that the optimal staffing policy is of "capture-all-demand" type and under the frame of this model for any unit staffing cost (even if it is very high) and for any cost saving potential (even if it is very low), it is optimal to invest in a nurse line. A sensitivity analysis of a symmetric case ($\pi = 0.5, c_A = c_B, C_A = C_B$) indicates that two performance measures, waiting time and error probability, are substitutes: there is no way to improve them both. As a matter fact, the patient does not get any positive surplus in equilibrium. Extensive numerical results are presented in the original paper to perform sensitivity analysis, please refer to it for your own interest.

Hospital staffing

Hospital staffing, i.e. how to allocate nurse and surgeon resource. de Vericourt et al. seeks to determine the fixed nurse-to-patient staffing ratio policy is effective with a predictive queueing analytic approach. It is assumed that there is a significant correlation between delays in addressing patient needs and averse medical outcomes and we promote using the frequency of excessive delay as a measure of staffing policy performance. By applying new many-server asymptotic results, we develop two heuristic staffing policies that perform very well and are easy to implement. This is the first many-server asymptotic analysis of health care issues. Among the insights gained from the heuristics is the realization that no ratio policy can provide consistently good quality of service across medical units of different sizes. Moreover, the optimal staffing levels for larger systems display a type of super pooling effect in which the requisite workforce is significantly smaller than the nominal patient load.

To briefly summarize, like traditional capacity management papers, the recent literature that optimizes the capacity management of limited resources, like operating rooms, diagnostic devices and medical-care staffs, with the help of stochastic and/or combinatorial optimization methods.

3.2 Appointment Scheduling

This subsection is also concerned with the optimization of capacity utilization, but with particular focus on the queueing systems with scheduled arrivals to heal-care institutions known as appointment systems. A better-designed appointment system can reduce waiting time for customers and increase the utilization of expensive personal and other resources. Thus, a significant part of the OR/OM literature is devoted to appointment scheduling of outpatient services, the first of which was Bailey (1952). For comprehensive literature reviews on appointment policies, we suggest Mondschein et al. (2003), or Cayirli et al. (2003) and Gupta et al. (2008), for, particularly, appointment scheduling of outpatients.

Response to appoint no-shows

An important aspect of customer behavior that influences the overall efficiency of such systems is the phenomenon of no-shows. Hassin et al. (2008) seeks to characterize and compute the optimal schedule with no-shows into consideration and identify whether no-shows is still costly with no-shows. The model assumes each patient's showing up probability p. We want to determine a schedule for a fixed number of patients in order to minimize the sum of expected patient's waiting costs and expected hospital's availability cost. Patients are served in the order of their scheduled appointments. The scheduled arrival of the kth customer is t_k, and if $t_i = t_j$, $i < j$, patients i and j will both show up, with i served before j. The service times are i.i.d of exponential distribution of mean μ^{-1}. Thus the model can be formulated as a $S(n,p)/M/1$ queueing system with n scheduled independent patients each showing up with probability p according to the schedule $S(n,p)$. The decision variable

$x = (x_1, x_2 \ldots, x_n - 1)$ is a vector of intervals between scheduled arrival times, so $t_1 = 0$, $t_i = \sum_{j=1}^{i-1} x_j$. We are now able to give the objective function:

$$\begin{aligned}\Phi(x) &= c_w p \sum_{i=1}^{n} w_i^s + c_s(\sum_{i=1}^{n-1} x_i + E[\text{server's time after } t_n]) \\ &= c_w p \sum_{i=1}^{n} w_i^s + c_s(\sum_{i=1}^{n-1} x_i + w_n^s + \frac{p}{\mu}),\end{aligned}$$

where c_w is the waiting cost c_s is the service cost, and w_i^s is the expected waiting time of the ith customer ($w_i^s = 0$). To evaluate the effect of no-shows, we only need to see the difference between $\Phi(x)$ and total expected service cost $c_s pn/\mu$. That is:

$$\Omega(x) = \Phi(x) - \frac{c_s pn}{\mu}.$$

We are still enabled to compute w_i^s analytically by recursion method, the details of which is shown in the original paper. Numerically experiments, provided in Hassin et al. (2008), illustrates, curiously, that both $\Phi(x^*)$ and $\Omega(x^*)$ increase first and decrease then as the showing up probability p rises from 0 to 1.

Green et al. (2008) also analyzes the no-show problem by conceptualizing the appointment system as a single-server queueing system in which customers who are about to enter service have a state-dependent probability of not being served and may rejoin the queue. Stationary distributions of the queue size are also derived, assuming both deterministic as well as exponential service times.

The model assumes that there is a finite queue length K, so that patients who arrive when the backlog is K are lost. The service times are deterministic with time T. So, the system is an $M/D/1/K$ queue. The assumption of fixed service times is consistent with our goal of providing guidance on patient panel sizes that result in short backlogs. Because physicians strive to see a fixed number of patients each day, idle time and delays during the day due to service time variability are inconsequential for this purpose. In reality, patients often change their plans while being on the waiting list, generating reschedulings and no-shows. In the case of an appointment rescheduling, we assume that a patient moves from his/her current place on the waiting list to its end and so does not affect the total length of the backlog. On the other hand, no-shows, or equivalently, last-minute cancellations, result in a service slot being unused. If a no-show patient does not reschedule, then the backlog dynamics are unaffected. However, many no-shows do schedule a new appointment and hence generate an additional demand. The probability of a no-show is a function of the length of the backlog at the time at which the patient makes an appointment, denoted as $r\gamma(k)$, where k is the length of the queue and r is a constant. Under the above mentioned setting, Green et al. derives the equilibrium distribution and demonstrates that the rescheduling probability have a significant impact on system performance and on the maximum patient panel size that can be reasonably handled.

Open-access policy

As a recently developed scheduling policy, open-access policies are also studied, by a comparison between them and traditional policies, in Robinson et al. (2010). Open-access policy, unlike the traditional policy under which a patient makes a routine appointment months ahead of time, allows a random number of patients call in the morning to make an appointment for that day. Both the traditional and the open-access scheduling policies encounter substantial variability in the number of patients seen per day: traditional because of no-shows within the fixed number of appointments for the day, and open-access because of the varying number of patients who call in to be seen each day. This paper makes the first attempt to compare the effects of these two types of variability

on the operational cost of the doctors office and to identify conditions under which each policy will be preferred. One of the major findings in Robinson et al. (2010) is that the open-access schedule outperforms the traditional schedule in the wide majority of cases. The traditional scheduling policy will be preferred only when the no-show probability is small (less than 5%) or the cost of patients waiting is trivial relative to the cost of the doctors time. Moving from a traditional schedule to an open-access schedule will allow the physician to increase the panel size (the number of patients on his or her books) by up to 30% when patient waiting is especially costly.The paper considers two open-access policies: (1) all patients must be seen the day they call in (the same day policy) or (2) some patients are willing to wait to be seen the following day (the same-or-next-day policy). The same-day policy will perform better for larger no-show probabilities, for larger workloads, and for smaller overtime surcharges. Also, the same-or-next-day scheduling policy is substantially less costly than the same-day policy only when the work day is approximately equal to the expected workload; otherwise, the option of deferring service makes little difference.

We consider the traditional appointment scheduling first, in which the length of time between appointments is the deterministic service time. Let T be the number of slots if appointments, while the number of patients scheduled for each day is N with show-up probability p. Hence, $\bar{n} = (1-p)N$ is the expected workload of patients a day. For the final patient N, his time slot scheduled is $t_{\max} = \max\{t : x_t \geq 1\}$. Define $b(k|z,\phi)$ be the probability mass function of binomial distribution $\binom{z}{k}(\phi)^k(1-\phi)^k$ and z_t be the possible of patients in the system in time t, so $z_t = (z_{t-1}-1)^+ + x_t$. Clearly, there will be two parts that contribute to the probability of k patients in the system during time t, $\pi_t(k)$: (1) the probability that the system was empty in the previous period and k patients arrive this period; and (2) the probability that there are $k+1-j$ patients in the system in the previous periods and j arrivals in this period. Therefore, $\pi_t(k)$ satisfies the following recursive equation:

$$\pi_t(k) = b(k|x_t, 1-p)\pi_{t-1}(0) + \sum_{j=(k+1-z_{t-1})^+}^{\min\{x_t,k\}} b(j|x_t, 1-p)\pi_{t-1}(k+1-j),$$

with boundary value: $\pi_0(0) = 1$ and $\pi_0(j) = 1$ for $j \geq 1$.

For a traditional schedule ($\{x_t\}$, denoting the number of patients scheduled for time t), its total cost $C^T(\{x_t\})$ consists of three parts: the doctor s expected length of the day ($\bar{D}^T$), the overtime time $\bar{O}^T$ and the waiting time $\bar{W}^T$. For the expected day length $\bar{D}$, the doctor must remain through the end of time slot $t_{\max} - 1$ to ensure all the patients' show-up. Hence,

$$\bar{D}^T = t_{\max} - 1 + \sum_{k=1}^{z_{t_{\max}}} k\pi_{t_{\max}}(k).$$

The idle time is the difference between the day length and the workload, so:

$$\bar{I}^T = \bar{D}^T - \bar{n} = t_{\max} - 1 + \sum_{k=1}^{z_{t_{\max}}} k\pi_{t_{\max}}(k) - \bar{n}.$$

The overtime calculated as the difference between the actual server completion time (D) and the length of the day T. As a result $\bar{O}^T = E_D(D-T)^+ = 0$ if $t_{\max} \leq T - z_{t_{\max}}$; $\bar{O}^T = \sum_{k=1}^{t_{\max}-z_{t_{\max}}-T-1} k\pi_{t_{\max}}(k+T+1-t_{\max})$ if $T - z_{t_{\max}} + 1 \leq t_{\max} \leq T$; and $\bar{O}^T = \bar{D}^T - T$ otherwise. To get the expected waiting time, we note in final period $t_{\max}$, each patient j in the system must wait for an additional $(j-1)$

periods before being served. Thus, the expected waiting time is:

$$\begin{aligned}\bar{W}^T &= \sum_{t=1}^{t_{\max}-1}\sum_{k=1}^{z_t}(k-1)\pi_t(k) + \sum_{k=1}^{z_{t_{\max}}}(\sum_{j=1}^{k}(j-1))\pi_{t_{\max}}(k) \\ &= \sum_{t=1}^{t_{\max}-1}\sum_{k=1}^{z_t}(k-1)\pi_t(k) + \frac{1}{2}\sum_{k=1}^{z_{t_{\max}}}(k-2)(k-1)\pi_{t_{\max}}(k).\end{aligned}$$

So, we have

$$C^T(\{x_t\}) = \bar{I}^T + \alpha\bar{W}^T + \beta\bar{O}^T,$$

where it is assumed that the cost rate of idle time is 1, of patients' waiting time is α and of overtime is β. The costs are clear: (1) $\sum_{t=1}^{T} = N$, (2) $\sum_{t=1}^{k} x_t \geq k$ for any k and x_t is a nonnegative integer. It is verified in the paper that the optimal policy contains no holes: if it is optimal to schedule a patient to period t, it will then be optimal to schedule patients for every earlier time slot.

We turn to take a look at the open-access policy now. The arrival distribution of requesting appointments is assumed to be a Poisson distribution of mean $\bar{n}$, thus of variance $\bar{n}$, which is bigger than that of the traditional case $Np(1-p) = p\bar{n}$. Let the cumulative distribution of the Poisson distribution be $P(s|\bar{n})$ and $\bar{P}(s|\bar{n}) := 1 - P(s|\bar{n})$. The model assumes the no-show rate to be zero for open-access scheduling. Thus, the optimal patient-sequencing problem is then trivial: assign patients sequentially to time slots, starting with the earliest in the day. This eliminates both idle time and patient waiting time; becomes irrelevant. Thus, the randomness in the demand arrivals will affect the system through overtime only. For a same-day scheduling, it is clear the expected overtime can be characterized as follows:

$$\bar{O}^{SD} = \sum_{s=T+1}^{\infty}(s-T)p(s|\bar{n}) = \bar{n}\bar{P}(T-1|\bar{n}) - T\bar{P}(T|\bar{n}).$$

Thus, the total expected daily cost:

$$C^{SD} = \beta\bar{O}^{SD}.$$

When demand is unusually high, the hospital defers some patients appointments to the following day. We can represent this extension as a Markov chain and a full description and analysis of it is given in the original paper. A strong dominance of the open-access policy to the traditional policy (the saving of expected cost is up to $40\% - 90\%$) is demonstrated by the numerical analysis in Robinson et al. (2010).

The no-shows and cancellations of patient appointments are also considered in Liu et al. (2010). It models the problem from the dynamic appointment scheduling decisions perspective. Extensive simulations were conducted to prove that the heuristics proposed outperform *all* other benchmark policies, particularly when patient is higher than regular capacity and that the open access policy, i.e. "meeting today's demand today", can be a reasonable choice when the patient load is low. An MDP model is proposed, but to get an optimal solution through policy improvement or value iteration is merely impossible due to the significantly high dimension. Hence, the paper gives a probabilistic static policy which is very close to the optimal policy.

Dialysis appointment

Patients suffering from a chronic condition often require periodic treatment. For example, patients with End-Stage Renal Disease (ESRD) require dialysis three times a week. These patients are also frequently hospitalized for complications from their treatment, resulting in idle capacity at the clinic. These temporary patient absences make overbooking at the clinic attractive. Lee et al. (2009)

develops a semi-closed migration network to capture patient flow into the clinic and between the clinic and hospital. The paper considers a simple class of stationary control policies for patient admissions and provide algorithms for selecting one that maximizes long-run average earnings. Local diffusion approximations were constructed to provide square-root loading formulas for the optimal capacity level and patient overbooking level: as the total patient arrival rate increases, the deviation between the optimal and fluid-limit capacity and overbooking levels scale up with the square root of the total arrival rate. We find that high hospitalization rates and long inpatient stays allow for more overbooking.

The stochastic facility model is a multi-class migration network with two nodes. Node 1 represents the clinic, and node 2 reflects temporary absences due to patient hospitalization. There are J classes of patients. The arrival process of all nodes are independent Poisson processes. If the total number of the total number of patients in the 2 nodes of the network is less than a decision threshold, M, a new patient is admitted. All admitted patients join node 1 (the clinic) first and receive treatment three times of a week. Once at that node, a class-j patient may move to node 2 (hospitalized), or department from the system, each in an independent exponential period of time. While at node 2, the patient may department the system, due to death, or return to the clinic, also each in an independent exponential period of time. The number of patients at time t in class j node i is $X_{ij}(t)$, so the total number of patients in node i is $X_i(t) = \sum_{j=1}^{J} X_{ij}(t)$. In addition to M, the maximum number of patients admitted into the clinic, the decision variable also includes the clinic capacity C. Let r be the margin for each patient treatment, and e be unit capacity cost per unit time and s be over treatment penalty. So we have the average long-run earning is:

$$AP(C, M) = \lim_{t\to\infty} \frac{1}{t}[r \int_0^t \min\{X_1(s), C\}ds - e \int_0^t Cdv - s \int_0^t (X_1(s) - C)^+ ds.$$

This objective function can be formulated by identifying the steady-state distribution, which can be found in details in the original paper.

For each patient threshold M, one can compute the optimal capacity limit C_M. Then, we can find the patient decision threshold that maximizes the long-run average earnings. Furthermore, because it can be shown that the steady-state distribution is monotone in M, the search for the optimal combination of M and C can be restricted to a finite set. Thus, the optimal control parameter C^* and M^* can be easily characterized. We can also reduce the capacity shortage probability by pooling capacity between facilities. The paper also establishes the optimal control and overbooking level by asymptotic approximation. The model is also demonstrated to be robust by the extensive numerical results presented in the paper.

To summarize, the literature that tries to address the optimal appointment scheduling issue mainly focuses on neutralizing the effect of no-shows and last-minute cancellations. A new dynamic policy called open-access to solve this problem is also broadly studied in recent papers, whose key is to allow a random number of calls with appointments scheduled each day.

3.3 Supply Chains in Health-Care

The interaction between supply management and health-care operations is widely studied in the recent literature. Supply chains in health-care system often have special properties like they must handled with high efficiency, their quality requirement is restrictive and the constraint and uncertainty at both yield and demand levels, so on and so forth. As has been discussed above, papers like Chick et al. (2008), Deo et al. (2009) and Cho (2010) all studies the influence of yield uncertainty in vaccination

production. They help us understand the strategic behaviors of vaccine manufacturers in response to yield uncertainty. In this subsection, we review a few latest papers that highlight supply chain models applied to health-care issues.

Reimbursement

Tackling the steep increase in drug costs is an especially important issue among many health care providers and insurers. To entice the clinics to become more cost efficient, the U.S. federal government, as well as many HMOs, have developed various cost containment initiatives recently. To formally analyze the impact of these initiatives on the patients well-being, the clinics profitability, and the pharmaceutical firms profitability, So et al. (2000) develops a model that examines the impact of a reimbursement policy for drug usage. This is the first few studies that emphasize the joint impact of the reimbursement policy on the patients, the clinic, and the pharmaceutical firm. The policy studied is an outcome oriented reimbursement which provides incentives for caregivers to offer appropriate patient care with effective use of resources.

Consider a clinic that purchases a drug from a pharmaceutical firm and prescribes this drug to a patient. The patient, with his well-being scored X_t at t, receives the drug treatments from the clinic (the dosage denoted as D_t) on a regular basis over an extended period of time. The response of the dosage follows the following linear relationship:

$$X_t = X_{t-1} + \epsilon_t + \alpha(D_t - D_{t-1}),$$

where ϵ_i's are i.i.d random normal variables with mean 0 and variance ϵ, representing some random fluctuation added to the patient and α is the response rate of the drug per unit dosage. The clinic applies the target prescription policy as follows: Based on the patients score X_{t-1} at the end of period $t-1$, the clinic would specify the subsequent dosage D_t that would adjust the patient's score X_{t-1} to a target level T (in expectation, i.e. $E(X_t|X_{t-1}) = T$). Hence, applying the expression of X_t and T, we get:

$$D_t = D_{t-1} + \frac{T - X_{t-1}}{\alpha},$$

and

$$X_t = T + \epsilon_t.$$

To provide a drug treatment according to the target prescription policy to a patient, the clinic has to pay cD_t to the drug manufacturer at the beginning of period t. After the treatment, the clinic files a claim with the patients insurer for reimbursement that covers the cost of the drug, various operating costs, and a profit margin (pD_t in total, $p > c > 0$). The insurer will accept the claim if the score of the patient is less than or equal to some threshold level K. Under this scheme, the clinics claim will be approved only when the patients well-being is below a certain threshold value K. To determine the optimal target level T^*, we just need to characterize the expected profit, R_t, for a patient at period t.

$$\begin{aligned} E(R_t) = E[E(R_t|D_t)] &= E[p(D_t P(X_t \le K)) - cD_t] \\ &= E(D_t)(pP(X_t \le K) - c) \\ &= \frac{T - X_0}{\alpha}(pP[X_t \le K] - c). \end{aligned}$$

Hence, T^* is identified by the following optimization problem:

$$\max_{L \le T \le U} E(R_t) = \max_T \frac{T - X_0}{\alpha}(pP[X_t \le K] - c),$$

where $[U, T]$ is the feasible range of T, and X_t, as shown above, follows the distribution $N(T, \sigma^2)$. It is also demonstrated in So et al. (2000) that T^* is increasing in the patient initial condition X_0, decreasing in the cost revenue ratio c/p, decreasing in the fluctuation σ^2 and increasing in the reimbursement threshold K, all in accordance with our intuition.

The reimbursement policy is also studied when dealing with ubiquitous purchaser-provider relation is also widely studied in recent literature. However, their objectives are often conflicting and there is one party enjoying information advantage. Fuloria, et al. (2001) develops a dynamic principal-agent model focusing on their interaction between this two parties. The model also seeks to understand effect of reimbursement policy. In this model, patients arrive exogenously, receive periodic treatment from the provider, suffer costly complications that require hospital care, and eventually exit the system in death. The provider chooses the intensity of treatment in each period, incurs an associated cost, and is reimbursed by the purchaser according to observed patient outcomes.

The dynamic principal agent model is based on the one presented in Plambeck et al. (2000). The model consists of a patient dynamics and a cash flow dynamics. It assumes Q_{t-1} patients requiring patient at the beginning of period t, and its wealth, necessary for the purposes of banking assumption, is denoted by W_{t-1}. In each period, the provider chooses a treatment decision $a_t \in \Omega = [\underline{a}, \bar{a}]$ and consumes an amount c_t. The cost of the provider incurs a cost $g(t)Q_{t-1}$, where $g(\cdot)$ is convex and increasing. The treatment intensity affects stochastically the number of patients who will suffer an expensive complication and will require hospital treatment in that period, denoted as H_t, and the number of death D_t. The distribution of H_t and D_t are assumed to be independent binomial variables with trial number Q_t, and probability parameter $p_H(a_t) = \bar{p}_H(a_t) = \bar{p}_H \exp(-\theta_H a_t)$ and $p_D(a_t) = \bar{p}_D(a_t) = \bar{p}_D \exp(-\theta_D a_t)$, respectively. The one-period long hospital treatment are conducted the next period by a third party and will return to the provider after treatment. Following that the purchaser receives a reward κ for each surviving patient and incurs a cost V_H per hospital treatment. She then makes a payment s_t to the provider according to:

$$s_t = s^0 + s^F Q_{t-1} + s^D D_t + s^H H_t,$$

where the payment coefficients are specified through contract negotiation. At the end of period t, a Poisson arrival A_t with rate λ of patients occur. Thus, the state the system becomes:

$$Q_t = Q_{t-1} - D_t + A_t,$$

and the provider's wealth becomes:

$$W_t = \frac{1}{\delta}(W_{t-1} - g(a_t)Q_{t-1} - c_t) + s_t,$$

where $\delta^{-1} - 1$ is the interest rate.

The model assumes the purchaser to be risk neutral, so it optimizes his infinite-horizon expected discounted social welfare, with discount factor δ:

$$E[\sum_{t=1}^{+\infty}(\kappa(Q_{t-1} - D_t) - V_H H_t - s^0 - s^F Q_{t-1} - s^H H_t - s^D D_t)|Q_0].$$

On the other hand, the provider is risk averse, and he seeks to optimize the expected utility from consumption, which exponential with constant absolute risk aversion rate r:

$$E[-\sum_{t=1}^{+\infty}\delta^{t-1}\exp(-rc_t)|W_0, Q_0].$$

The provider requires his utility be no less than U. All information except for provider's own action and consumption are observable to two parties. The purpose of this model is to find the optimal treatment and consumption strategy in response to payment contract (s^0, s^F, s^D, s^H), and to find out the optimal contact. An important contribution of the analysis in Fuloria et al. (2001) is a dichotomy of incentive payments into short term and long term: the outcome-adjusted-system motivates the provider to deliver the desired treatment by penalizing single-period adverse outcomes and rewarding patient longevity. For analytical details and numerical illustrations, please refer to the original paper.

We shortly introduce a few other papers of modeling novelty but not focusing on analytical approach. Supply limitations are very common in drug industry. As a result, forecasting supply-constraint drug demand in markets is difficult since the supply limitation have significantly curtailed sales volumes and thus reduced the usefulness of conventional sales-based forecasting methods. Stonebraker et al. (2009) applies a decision analytical model to explicitly track variability in epidemiological data together with the variability in treatment modalities to estimate latent therapeutic demand (LTD)the underlying demand that captures how physicians would prescribe treatment and how patients would comply if ample supplies of drugs were available and affordable. THirumlai et al. (2011) performs an econometric analysis to investigate the persistent quality and the induced product-recall problems of medical devices. The study aims to assess the financial implications of medical device recalls, particularly in the aspect that whether this consequence is so severe that it defers firms from introducing potentially hazardous medical devices into the market. This empirical research suggests that medical recalls bring significant discomfort to consumers but it causes little capital market penalties. The paper also demonstrates that the likelihood of recalls decreases with the cumulative recall experience of firms, i.e. there is a learning effect from improvements to firm operations induced by former recalls. Besides suffering from uncertain patient arrivals, primary-care clinics also face the uncertainty in patient choices. Patients have different perceptions of the acuity of their need, different time-of-day preferences, as well as different degrees of loyalty toward their designated primary-care provider (PCP). The clinic needs to decide which appointment requests to accept to maximize revenue. The paper develops a Markov decision process model for the appointment-booking problem in which the patients choice behavior is modeled explicitly. When the clinic is served by a single physician, we prove that the optimal policy is a threshold-type policy as long as the choice probabilities satisfy a weak condition.

3.4 Other Issues not Listed

Revenue management in non-profit operations

Nonprofit firms sometimes engage in for-profit activities for the purpose of generating revenue to subsidize their mission activities. The organization is then confronted with a consumption versus investment trade-off, where investment corresponds to providing capacity for revenue customers, and consumption corresponds to serving mission customers. de Vericourt et al. (2009) models this problem as a multi-period stochastic dynamic program. In each period, the organization must decide how much of the current assets should be invested in revenue-customer service capacity, and at what price the service should be sold. We start with the resource allocation problem with banking. We call R-customers the customers for generating revenues and M-customers the ones not charged of any fee.

For a medical service institution with assets a_t at period $t \in \{1, 2, \cdots, T\}$, the institution needs

to decide how much capacity y_t to provide for R-customers and how much, z_t to be banked. Thus, $x_t = a_t - y_t - z_t$ units of asset are allocated to M-customers. The demand of R-customers is θ_t, thus the number of R-customers served is $y_t \wedge \theta_t$. So, the resources at the beginning of the next period is:

$$a_{t-1} = p(y_t \wedge \theta_t) + \beta z_t + \Delta_t,$$

where p is the profit per R-customer and Δ_t is the random variable denoting the donation at t. Let c_R and c_M be the cost of serving a R-customer and M-customer, respectively, while their social impact return is τ and s. We assume s/c_M to be 1 and, thus, $\tau \leq 1$. As a result, the total social return is $(a_t - y_t - z_t) + \tau(y_t\theta_t)$. The model considers the finite horizon, T, condition, so the maximum social-impact-to-go function $v_t(a)$ at period t with resource a can be characterized as follows. Clearly, at period T, all assets are allocated to serving M-customers. Hence, $v_T(a) = a$. We have:

$$v_t(a) = \max_{0 \leq y+z \leq a} (a - y - z) + \tau E_y(y \wedge \Theta) + \alpha H^{v_{t+1}(y,z)},$$

where $H^v(y, z) = E_\Delta E_y(v(p(y \wedge \Theta) + \beta z + \Delta))$ with Δ following the distribution of Δ_t, Θ following the distribution of θ_t and α as the social discount factor for delaying service to an M-customer to the next period. The optimal policy for this capacity allocation and banking problem is of threshold type: capacity should only be allocated to M-customers when assets are above a threshold, and all assets above that threshold should be allocated for this purpose.

The paper also discusses the optimal resource allocation with pricing. It considers a condition where the demand function of R-customers D_t is with multiplicative uncertainty, i.e. $D_t = \gamma(p_t)\Theta_t$, where $\gamma(\cdot)$ is the price response function. Intuitively, the optimal price should be decreasing in capacity allocated to R-customers. However, a simple counterexample is provided in the paper, but it still provides a sufficient condition for this intuition to be true. For more details about the modeling, analysis or numerical examples, please kindly refer to the original paper.

Impact of discharge decisions

The impact of discharging decisions in the face of shortages is investigated in Berk et al. (1998). It develops a model that elucidates the dynamics of a health care unit. To capture the essence of discharge decisions, the model considers discharge policies that incorporate both the occupancy level of the unit and the status of patients measured by their stage of recovery and the time they have spent in that stage. It is found in the paper that inclusion of early discharge option improves system accessibility significantly and does not jeopardize care equity among patients. Furthermore, introduction of early discharge option has more pronounced effects on increasing care unit capacity than addition of open beds with no early discharges.

Some other issues concerning the optimization of decisions conducted by institutions are also studied in recent literature. Crama el al. (2008) analyzes the optimal design of licensing contracts with incomplete information on the valuation of an innovation and limited control of licensee's development effort. The findings in this paper inform managerial practice about the advantages and disadvantages of milestone payments and royalties and recommend the optimal composition of the contract. Kc et al. (2009) applies an econometric model to analyze the impact of workload and service time on patient safety. It shows that workers accelerate the service rate as load increases. It has also been demonstrated that such acceleration may not be sustainable: long periods of increased load (overwork) have the effect of decreasing the service rate. The paper also investigates cadiothoracic surgery and indicates that an increase in overwork results in an increase in the likelihood of mortality. Estimating ambulance travel times is a key step to measure the performance of medical emergency

response. Budge. et al. (2010) applies nonparametric estimates of median and variation to this issue and finds that the coefficient of variation decreases with distance. The resulting travel-time distribution model can help us create probability-of-coverage maps for diagnosis and improvement of system performance.

4 Individual Patient & Disease Treatment Optimization

In this section, we are going to present papers that contribute to the understanding of and treatment diagnosis optimization from the perspective of patients. The diagnosis optimization literature usually aim to unearth the connection between symptoms and diseases, as well as to seek an optimal screening policy so as to detect and prevent certain diseases, like breast cancer. As for papers devoted to treatment optimization, they mainly consider the optimization of some particular therapies like radiation therapy chemotherapy and dialysis.

4.1 Diagnosis Optimization

The role of diagnosis is to connect symptoms with internal diseases in order to efficiently treat or prevent in advance. This part of literature usually applies statistical and computational tools to classify and stimulate the symptom/disease dynamics. There are also plenty of papers contributing to optimal screening policies.

Symptom classification and simulation

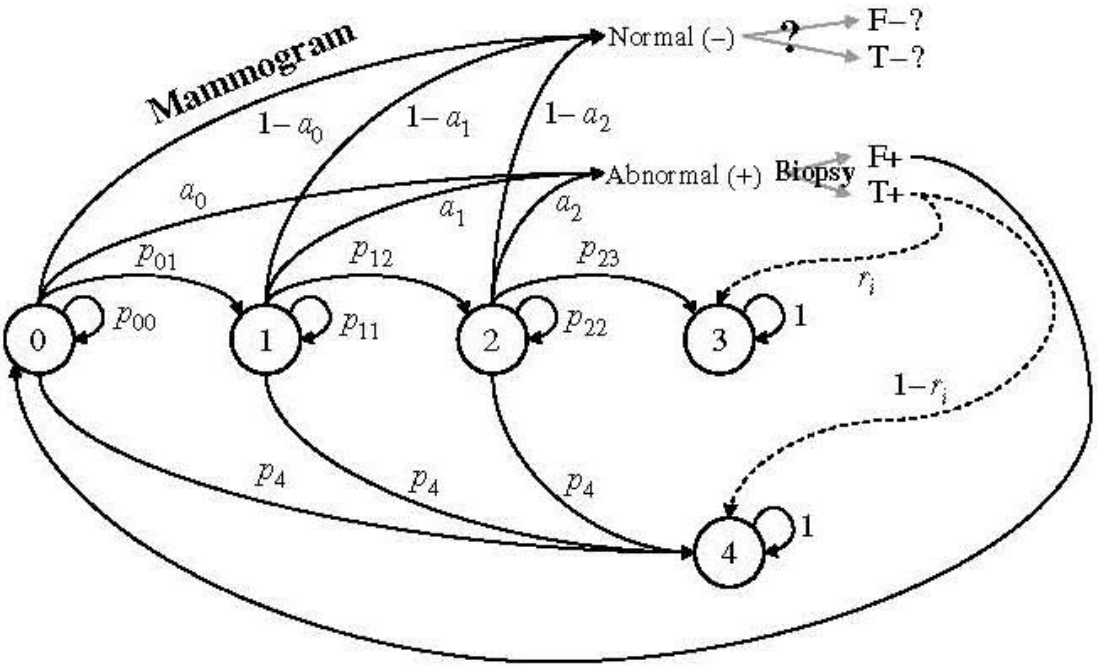
For many diseases, to determine what information to gather on symptoms and what combination of symptoms lead to a given disease is a great challenge in diagnosis. To generate adequate statistical data, the required number of experiments is usually unmanageably large. Hence, Saaty et al. develops an analytical hierarchy process (AHP) model where statistical data and expert judgment can be incorporated. Bayes theorem is utilized in this model to link posterior probabilities with experiment outcomes.

An explosion of interest in data mining and optimization research occurs recently to quantitatively discover and investigate the complex patterns in the vast amount of information generated by brain functions. Chaovalitwongse, et al. (2008) proposes a new classification technique, support feature machine (SFM), to classify abnormal brain activities. The SFM is essentially an optimization model that maximizes classification accuracy based on nearest-neighbor classification. The performance of this classification technique outperforms others, achieving, on average, over 90% classification accuracy.

There is also some literature that simulates the homeostasis of human body. Karanfil et al. (2008) builds a dynamic model to study the water regulation of human body, with concentration on the feedback mechanisms involved in their normal and abnormal physiology. Therapeutic interventions like intoxication and hyponatremia are also included in the simulation model. This system dynamical approach is proved to adequately reveal typical dynamics of the body by experimental data. The model includes a differential equation system that projects the normal and abnormal regulation of water. This simulator also allows physicians to perform experiments to test the effects of typical set of treatment options on a simulated patient.

Optimal screening policy

A great many of patients choose to participate in disease screening in the hopes of detecting diseases before they are outwardly observable. The relative value and frequency of mammography



Notes. In this rendition of the model, the parameters' dependency on age is suppressed. The outcome of each mammogram depends on the (unknown) underlying disease state: 0 (cancer free), 1 (early disease), or 2 (late disease). Following a false positive mammogram result, the patient reverts to state 0 with probability one. Following a true positive mammogram result, the patient exits the model and is eventually absorbed into state 3 (breast cancer death) with probability r_i, where i denotes the state upon detection, or state 4 (death from other causes).

Figure 3: Partially Observed Markov Chain Natural History Model

screening for pre-menopausal women versus post-menopausal women draws OR researchers' attention due to the conflicting age-based dynamics of both the disease (increasing incidence, decreasing aggression) and the accuracy of the test results (increasing sensitivity and specificity). Maillart et al. (2008) develops a partially observable Markov chain model to optimize the screening policy measured by a lifetime cancer mortality risk metric and an expected mammogram count. The analysis of this model demonstrates that screening should start relatively early in life and continue relatively late in life regardless of the screening interval(s) adopted.

The Markov chain model classifies the breast cancer progression as state 0 to state 4, according to its severity: 0 is no cancer, 1 is early breast cancer, 2 is advanced breast cancer, 3 is breast cancer induced death and 4 is non-breast cancer induced death, and 1,2,3 are in between. From stage j, $j = 0, 1, 2$, a patient of age α transits to state j with probability $p_{jj}(\alpha)$, to state $j+1$ with probability $p_{j,j+1}(\alpha)$ and to state 4 with $p_4(\alpha)$ (Figure 3). Let $P(\alpha)$ be the corresponding one-step transition probability matrix.

It is assumed that even if a patient develops symptoms, the earliest she can be diagnosed is at the time of her next screening. If the patient is in state j, then the result of a mammogram is either "abnormal", with probability $a_j(\alpha)$, or "normal" with probability $1 - a_j(\alpha)$. Since transitions between states 0, 1, and 2 are not outwardly visible, and because mammogram results may be false, the disease state is partially observed. Define the $\pi = (\pi_0, \pi_1, \pi_2)$ as the distribution over disease states, where π_j is the probability the patient is currently in state j. The performance of a screening policy is measured by the probability a patient will die from breast cancer under that policy. So the cost of state 3 is 1 and of state 4 is 0.

Let $W_n(\pi)$ be the probability that a patient will eventually die from breast cancer if she starts with occupancy distribution π, in period n at age α_n. If a screening policy does not prescribe a mammogram, the patient in the next period can either: (1) die from breast cancer with probability $\pi_2 p_{23}(\alpha_n)$ and cost 1; (2) die from another cause, with cost 0; or survive, with probability $1 - \pi_2 p_{23}(\alpha_n) - p_4(\alpha_n)$ and conditional distribution $\pi'(\pi)$ which follows Bayes' rule. Thus, under this condition, we have:

$$W_n(\pi) = \pi_2 p_{23}(\alpha_n) + [1 - \pi_2 p_{23}(\alpha_n) - p_4(\alpha_n)] W_{n+1}(\pi'(\pi)).$$

In the condition that the screening policy prescribes a mammogram in period n, the approach is similar but the Bayes' rule applies differently. The paper also conducts extensive sample path analysis of this Markov chain and shows the robustness of optimal policies. Rauner et al. (2010) also investigates the Pareto-optimal screening strategies against breast cancer. The model in this paper provides policy-makers Pareto-optimal screening schedules for risk groups by considering cost and effectiveness outcomes as well as budget constraints. Pareto ant colony optimization algorithm for multi-objective combinatorial optimization problems is utilized to derive the meta-heuristics solution technique.

4.2 Treatment Optimization

The final part of our review will be devoted to the literature that optimizes the effect of treatment. We discuss disease therapy initiation for HIV and end-stage renal failure first and then conclude this section with the optimization of radiation therapy and chemotherapy.

Disease therapy

The aim of this part of literature is to determine the optimal treatment policy from the perspective of patients. Shechter et al. (2008) investigates the optimal time to initiate HIV therapy. Benefits of delaying therapy include avoiding the negative side effects and toxicities associated with the drugs, delaying selective pressures that induce the development of resistant strains of the virus, and preserving a limited number of treatment options. On the other hand, the risks of delayed therapy include the possibility of irreversible damage to the immune system, development of AIDS-related complications, and death. A Markov decision process is developed to maximize the expected lifetime or quality adjusted lifetime of a patient.

The assumes that the patient monthly visits a physician periodically to check his disease state and to decide when to initiate therapy. For the health state of the patient s, which ranges from 1 to N, there will be a reward $r(s)$ the patient receives when waiting in state s and a expected total remaining reward, received when the patient initiates therapy from state s. We assign $r(0) = R(0) = 0$. The action taken by the patient at each period is either to wait or to initiate. If the patient chooses to wait in state s, the probability he will transit to state j is $p(j|s)$. Therefore, the MDP problem can

be characterized as:

$$v(s) = \max\{r(s) + \sum_j p(j|s)v(j), R(s)\},$$

where $v(s)$ is the optimal measure of the policy when the initial state is s and if we seek to optimize the quality adjusted lifetime of the patient, $r(s) = 1$. It is derived in the original paper that $v(s)$ is non-negative and decreasing in s. The optimal policy will be to initiate immediately whenever the reward of initiation $R(s)$ is big enough, $(R(s) \geq r(s) + \sum_j p(j|s)R(j))$. Numerical experiments based on clinical data, contradicts the recent trend to treat it late.

The treatment initiation problem also arises from dialysis therapy, which is the most common way for patients afflicted with chronic kidney failure. Lee et al. (2008) makes some attempt to understand the relation ship between dialysis initiation and therapy's cost and effectiveness. The paper builds an approximated dynamic programing model and computationally derives the optimal strategy. The numerical experiments in this paper show that: (1) standard early initiation strategies, where once started on dialysis patients are kept on a fixed weekly program, have a limited potential; and (2) dynamic strategies incorporating patient-specific characteristics to customize dosage can yield a significant cost advantage.

Radiation and chemotherapy

Cancer is a common disease of modern people. There is a huge number of patients diagnosed with cancer each year. Radiation therapy and chemotherapy are two of the major approaches against cancers, especially when the tumor is too big for surgical excision alone. However, their side-effect might be very big since both X and γ rays, for radiation therapy, and toxic drugs , for chemotherapy, also have great damage on healthy tissues. That is the reason why this area calls for optimization method so as to improve the efficacy and control the side effect of radiation and chemotherapy.

Recently, a huge number of cancer patients benefit from conformal radiation therapy. However, many patients that are initially considered curable do in fact die of their disease, despite sophisticated treatment. Others may suffer from unintended side effects from radiation therapy, severely reducing the quality of life. This happens mainly because radiation therapy plans often deliver too little dose to the targets but too much to health organs, or both. Intensity-modulated radiation therapy (IMRT) is considered to be able to balance the preservation of healthy tissues and the probability of eradication of the tumors. In IMRT, the patient is irradiated from several beams, each of which is decomposed into hundreds of small beam-lets, the intensities of which can be controlled individually. Romejin et al. (2006) considers the problem of designing a treatment plan for IMRT when the orientations of the beams are given. It proposes a novel linear programing approach to this problem, thus substantially improving the tractability of the optimization compared with the established mixed-integer and nonlinear programing approach.

Suppose the targets $s = 1, 2, \cdots, S$ and critical structures (i.e. the nearby healthy tissues) $s = S + 1, \cdot, T$ are considered. Each structure contains several v_s voxels. There are totally N beam-lets and the dose received by voxel j in structure s from beam-let i at unit intensity is D_{ijs}. Hence, the dose received by each voxel as a function of beam-let intensity $x = (x_1, \cdots, x_N)$ is:

$$D_{js}(x) = \sum_{i=1}^{N} D_{ijs}x_i.$$

The minimum prescription dose to all voxels in the target s is L_s and the maximum tolerance dose of both target and critical voxels is U_s. Hence, we have $D_{js}(x) \geq L_s$ for $s \leq T$ and $D_{js}(x) \leq U_s$ for $s \leq S$. Suppose there are under-dose threshold $T_s^U \geq L_s$ and over-dose threshold $T_s^O \leq U_s$. The penalty

functions for under-dose and over-dose are $F_s^U(z) = \beta_s^U[(T_s^U - z)^+]^{p_s^U}$ and $F_s^O(z) = \beta_s^O[(z - T_s^O)^+]^{p_s^O}$, respectively, where $\beta_s^U, \beta_s^O \geq 0$ and $p_s^U, p_s^O \geq 1$ to ensure convexity. The objective is to minimize:

$$F(x) = \sum_{s=1}^{S} \frac{1}{v_s} \sum_{j=1}^{v_s} F_s(D_{js}(x)).$$

The scaling $1/v_s$ is to ensure the penalty is insensitive to relative sizes of the targets and critical structures. To sum up, the optimization problem can be formulated as:

$$\min_x F(x) = \sum_{s=1}^{S} \frac{1}{v_s} \sum_{j=1}^{v_s} F_s(D_{js}(x)),$$

subject to, for alli, j, s

$$D_{js}(x) = \sum_{i=1}^{N} D_{ijs} x_i,$$

$$F_s(z) = F_s^U(z) + F_s^O(z) = \beta_s^U[(T_s^U - z)^+]^{p_s^U} + \beta_s^O[(z - T_s^O)^+]^{p_s^O},$$

$$D_{js}(x) \leq U_s,$$

$$D_{js}(x) \geq L_s,$$

$$x_i \geq N.$$

The paper also considers the commonly employed cumulative dose-volume histogram constraint (DVH), which specifies, for a given target or critical structure, the fraction of its volume that receives at least a certain amount of dose. Spatial correlations of different targets and critical structures are also investigated in it. We strongly suggest you refer to the original papers for their details.

When conducting the IMRT therapy to liver and lung cancers, the motion uncertainty induced by breathing is a challenge to the effective and reliable deliver of the radiation. In Bortfeld. et al. (2008) the authors build a probabilistic model of motion uncertainty and provides a robust formulation of the IMRT optimization problem. The major extension of this model compared with the one presented in Romejin et al. (2006) is that the radiation dose deliver intensity is D_{ijs} no longer a constant vector, but a random vector whose distribution changes cyclically with the breath cycle. This generalization of nominal approach (when the motion knowledge is certain and complete) performs well in the sense of maximum over-dose (within 11%).

In IMRT treatment, after the intensity profile is selected to both ensure targets receive sufficient dose and functional tissues are spared, the profile must be decomposed into a collection of apertures and corresponding intensities. Taskin et al. (2010) investigates this decomposition problem by an integer programing approach. An intensity profile is represented as a nonnegative integer matrix; an aperture is represented as a binary matrix whose ones appear consecutively in each row. A feasible decomposition is one in which the original desired intensity profile is equal to the sum of a number of feasible binary matrices multiplied by corresponding intensity values. To most efficiently treat a patient, the objective is to minimize a measure of total treatment time, which is given as a weighted sum of the number of apertures and the sum of the aperture intensities used in the decomposition. The paper has described an exact decomposition algorithm based on an integer programing model for finding multi-set of intensity values to be assigned to apertures and a backtracking algorithm that forms apertures by finding compatible leaf positions of each row. Computational results based on clinical data show that the method is capable of solving to optimality within a few minutes. You may consult the original paper for a complete description of the model, method and numerical experiments therein.

In order to ameliorate the chemotherapy treatment, Agur et al. (2006) identifies two general categories of anticancer drug protocols: a one-intensive treatment or a series of non-intensive treatments. Simulating the patients pharmaco-dynamics in a simple model for cell population growth, the paper calculates the number of drug susceptible cells at every moment of therapy. The problem, shown to be NP-hard, can be defined as an optimization problem to which the possible solutions are scheduling plans represented by zeros (no treatment) and ones (treatment). As a result, the authors tries three local search heuristics simulated annealing (SA), threshold acceptance (TA) and old bachelor acceptance (OBA) to search a desired a solution. Numerical results in this paper have demonstrated the competitiveness of the three heuristics but the computational effort of SA is higher than the other two.

In conclusion, papers that contribute to optimizing radiation therapy or chemotherapy usually attempt to balance the trade off between damaging cancer cells and sparing nearby healthy ones. Particularly, almost all recent research on radiation focuses the newly proposed IMRT optimization, which can be reformulated as a linear programing or related optimization model.

5 Future Research

The operations research and operations management method applied to health-care issues will continue to be a fruitful research area. As in the past, research will progress in many directions. The list below only reflects the author's knowledge and taste, which is inevitably incomplete.

Equity in resource allocation

Equity must be taken into consideration in the allocation of limited medical resources due to ethical concerns. As is discussed in Su et al. (2004), Su et al. (2006) and Sandikci et al. (2008), the policy that maximizes the system-wide performance may substantially sacrifice equity. This type of problem especially takes place in organ transplantation issues, where donated organs can only supply a fraction of patients waiting for transplantation surgery. The trade-off between the value of the information indicating health-condition of the patients in the waiting list and their privacy is surely a central problem that arouses both the organization and the patients' interest. There is still a long way to go before we can embed equity consideration quantitatively into the objective function that reflects the efficiency or revenue of a health-care policy.

The quantification of medical institution/policy's performance

One possible approach is the social-impact-adjusted revenue management of a medical institute, as is done in de Vericourt et al. (2009). However, in practice, the social impact of a policy can hardly be estimated precisely in terms of dollars. For example, how much do you equate enabling a cancer patient live more than 10 years with? Another related issue is that how should a government allocate health-care resources, such as fundings and vaccines, into different regions of the country. Like the discussion in Sun et al. (2009), the optimal transition of drugs, and other medical resources, is a central issue in the government's medical resource planning policy. Additionally, it is also a challenging job to incorporate the risk-aversion property of a government into the objective function, since the resulting function might be very hard to analyze.

Emergency response

At the institution level, emergency response should be conducted by the emergency department. Problems like optimal allocation and scheduling of the ambulance in a city to minimize its traveling time, as discussed in Budge et al. (2010), is very essential. To better react to emergencies, we still

need to analyze the optimal staffing and scheduling nurses so as to both shorten the operational time and improve the quality of emergency medical treatment. At the government level, as is analyzed in Craft et al. (2005), the response to nation-wide disasters, such as bio-error attacks, floods and earthquakes, that requests urgent medical services like the distribution of vaccine and drugs, the new construction of hospitals and the reallocation of health-care workers. Still, the ethical issues under such circumstances are also very intriguing but difficult.

Insurance

This is another research area that I think deserves our attention. Health insurance industry is in close connection with health-care systems. There interactions have great impact on the risk and revenue of insurance companies and hospitals as well as the cost of patients. For an insurance company, its task is to design health insurance optimally so as to reduce its risk of medical claims and attract as many patients as possible to buy their products. For a hospital, it, as shown in So et al. (2000) and Fuloria et al. (2001) that considers the optimal outcome-adjusted reimbursement policy, needs to perform treatment policies optimally in order to transfer part of the risk to insurance companies and to ensure the treatment effect on patients. For a patient, he has to choose suitable insurance products so that he will be able to pay for the treatment that leads him to recovery. It is also interesting to investigate the game and information asymmetry in their interactions and how to provide incentives so that they can coordinate to reach the system-wide optimality.

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